SPATIAL VOTING (MULTIPLE DIMENSIONS)

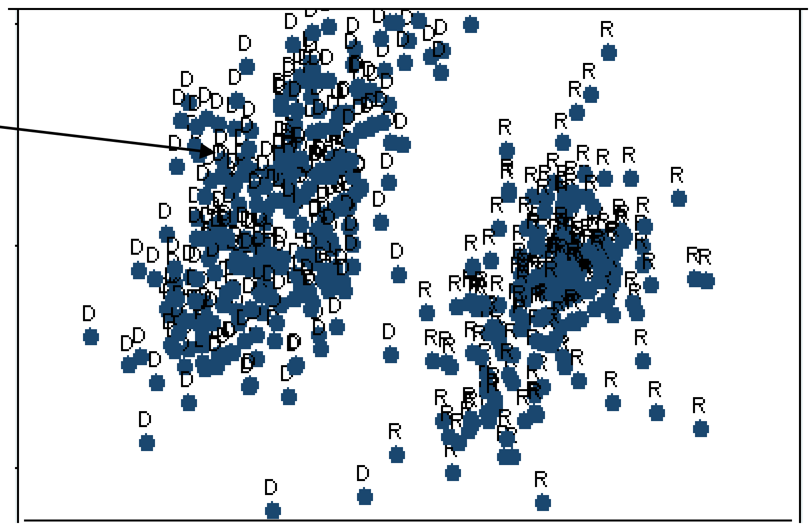
Assumptions

- Alternatives are points in an n-dimensional space.
  - Examples for 2D:
    - Social Issues and Economic Issues
    - Domestic Spending and Foreign Spending
- Single-peaked preferences
  - Each agent has an **ideal point** (most-preferred alternative)
- Symmetric preferences
  - Utility **declines** as a **distance from ideal point increases**

Congress

103rd House. Clinton’s First Term

Bill
Clinton



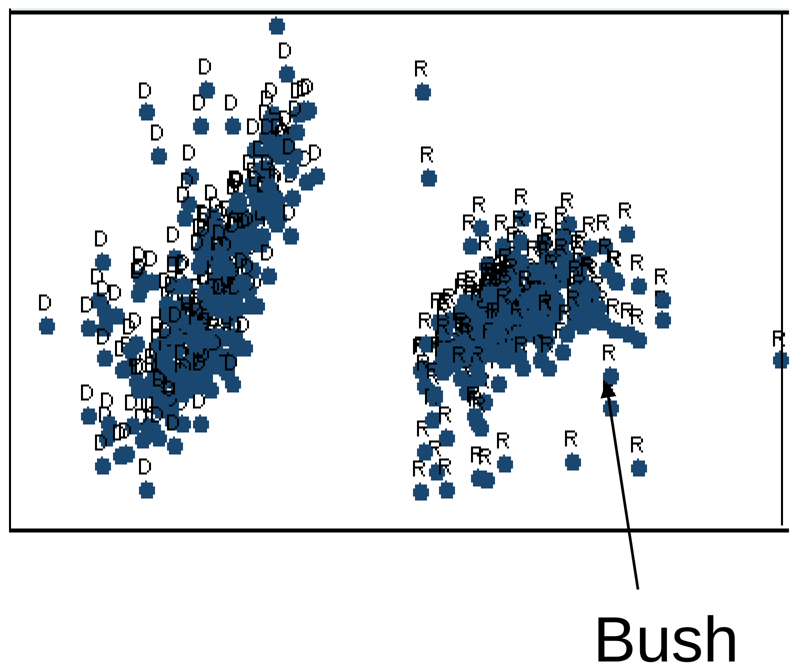
Source: DW-NOMINATE scores.

<u>Notice:</u>

Two distinct parties moderately spread apart.

Congress

110th House. W. Bush's last Term



Notice:

Two distinct parties very far apart.

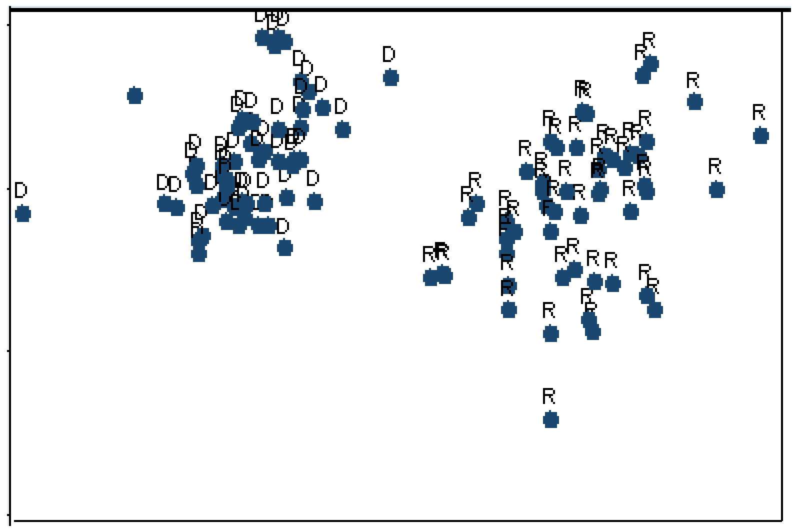
This suggests an increase in partisanship – at least in the House.

Congress

110th Senate. Prior to 2008 Presidential Election

The final three presidential candidates were all in the Senate.

Can you guess there locations?



Assumptions

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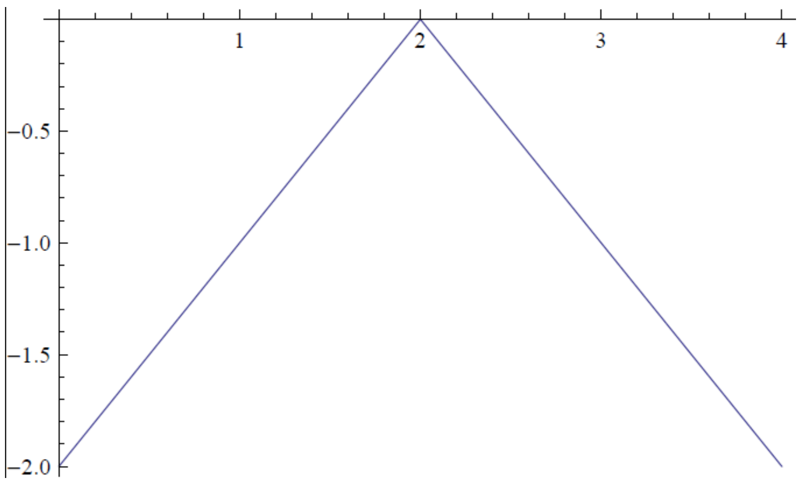
Single Dimension

Utility is a decreasing function of **distance** between the alternative and ideal point

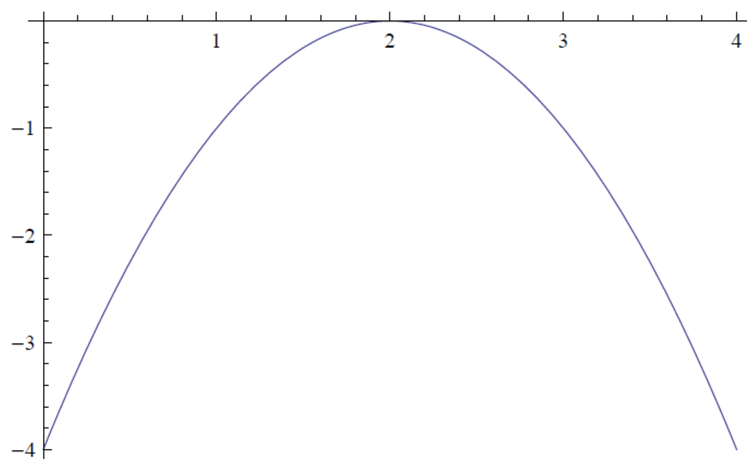


Linear (absolute value)

$$U(x) = -|x-2|$$

Quadratic

$$U(x) = -(x-2)^2$$



Two or more dimensions

Linear

$$U(x) = -\sum_{j=1}^{k} \alpha_j \left| x_j - \theta_j \right|$$

Quadratic

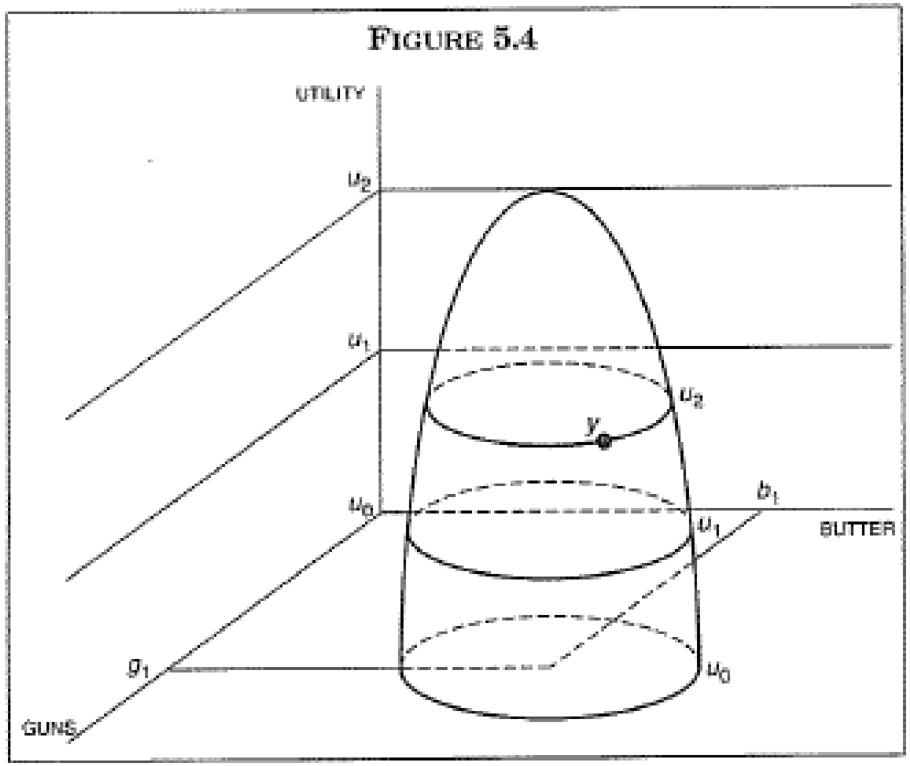
$$U(x) = -\sum_{j=1}^{k} \alpha_j \left(x_j - \theta_j \right)^2$$

j indexes dimensions

α_j = weight on dimension j

θ_j = ideal policy on dimension j

Two dimensions



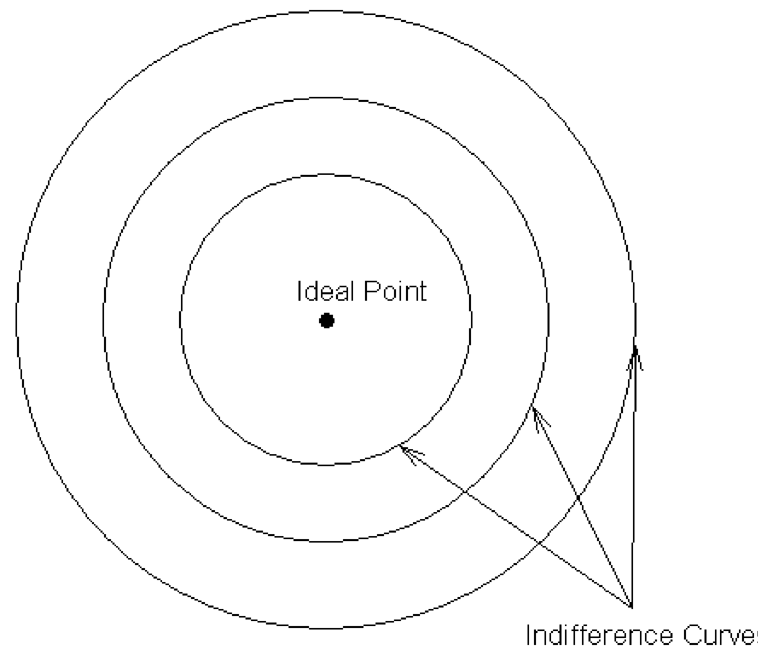
$$U(x,y) = -(x-\theta_1)^2 - (y-\theta_2)^2$$

Projection onto policy space

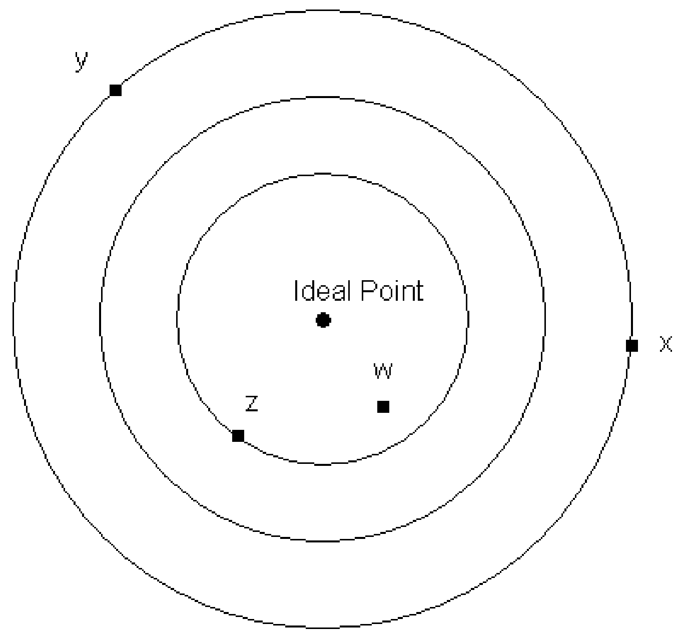
Indifference curve = set of points an individual is indifferent between

Example

$$U(x,y) = -(x-\theta_1)^2 - (y-\theta_2)^2$$



Projection onto policy space



w P z P x I y

Example

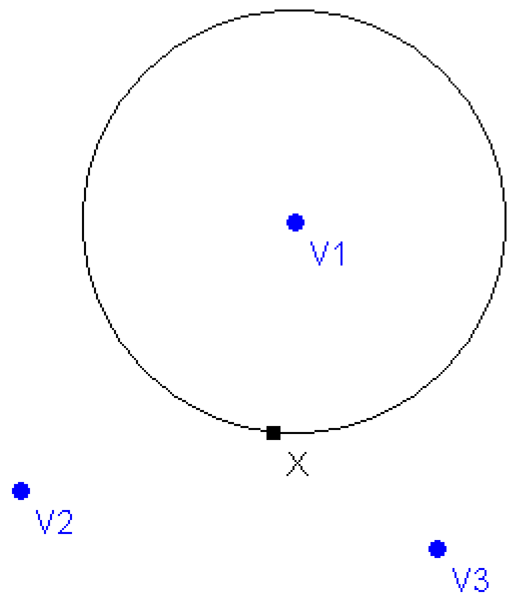
Assume two dimensional spatial model with single-peaked and symmetric preferences. For an individual with an ideal point at (2,1), rank order the following alternatives: (3,1), (0,0), (1,-1).

Extra Credit

- Each of you will be assigned an "ideal point" on a 10 X 10 square inclusive (randomly drawn out of a hat).
- We will vote on pairs of alternatives on the square using majority rule.
  - When we start, there will be no alternative on the floor (no one gets any points if there is no alternative on the floor when we adjourn).
  - Anyone can propose an alternative. For example, you may propose (50, 0) or you may propose (10.2, .81). If someone seconds, we will vote.
  - The alternative that receives a majority of votes becomes the number on the floor in the next round.
  - Voting concludes when someone motions adjournment, the motion is seconded, and a majority of players vote to adjourn. That number on the floor wins.
- Payoffs. You will receive $10 - (|x_1 - i_1| + |x_2 - i_2|)$ points on the final exam, where (x_1, x_2) is the winning number and (i_1, i_2) is your ideal point.
- In other words, the closer the final outcome is to your ideal point, the more points you receive.

Write your name on your number and give it to me.

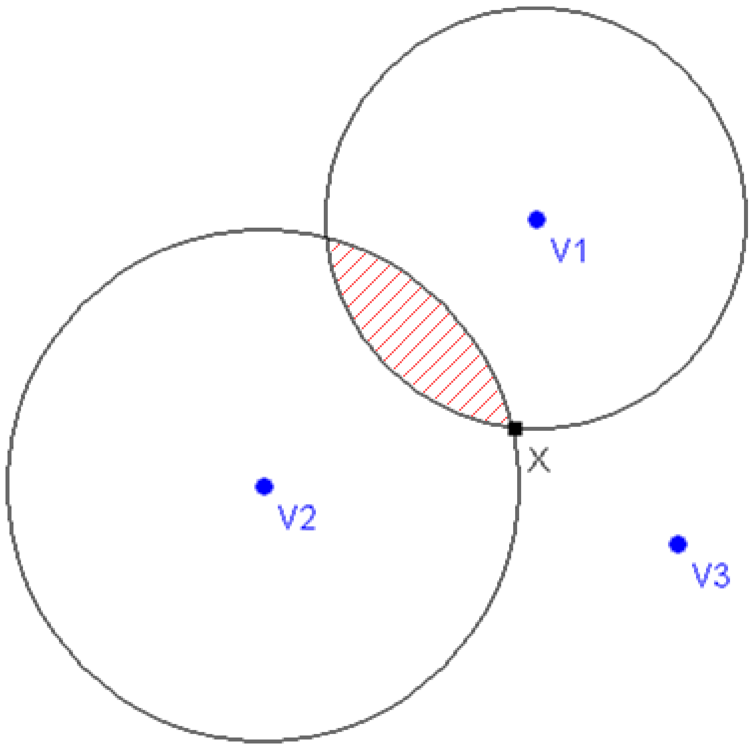
Win Set



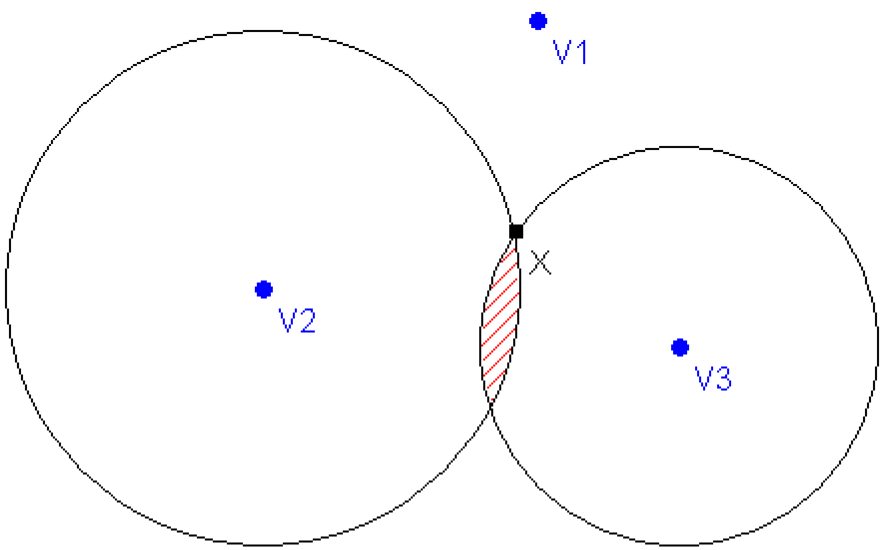
Recall: the majority rule win set of x is the set of alternatives that a majority prefers to x: W(x) = {y | yPx}

Created by drawing indifference curves through x and shading points that a majority of voters prefer to x.

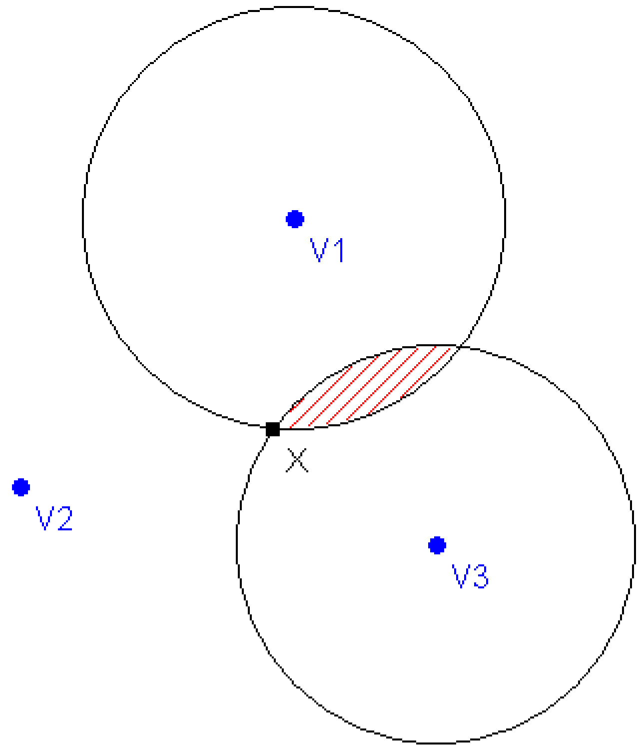
Win Set



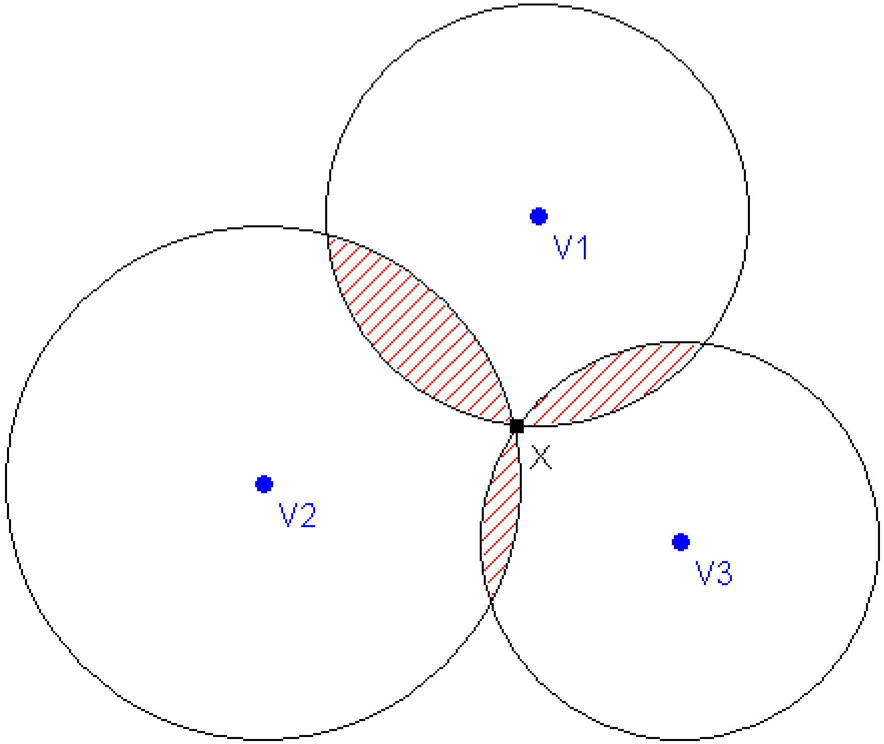
Win Set



Win Set

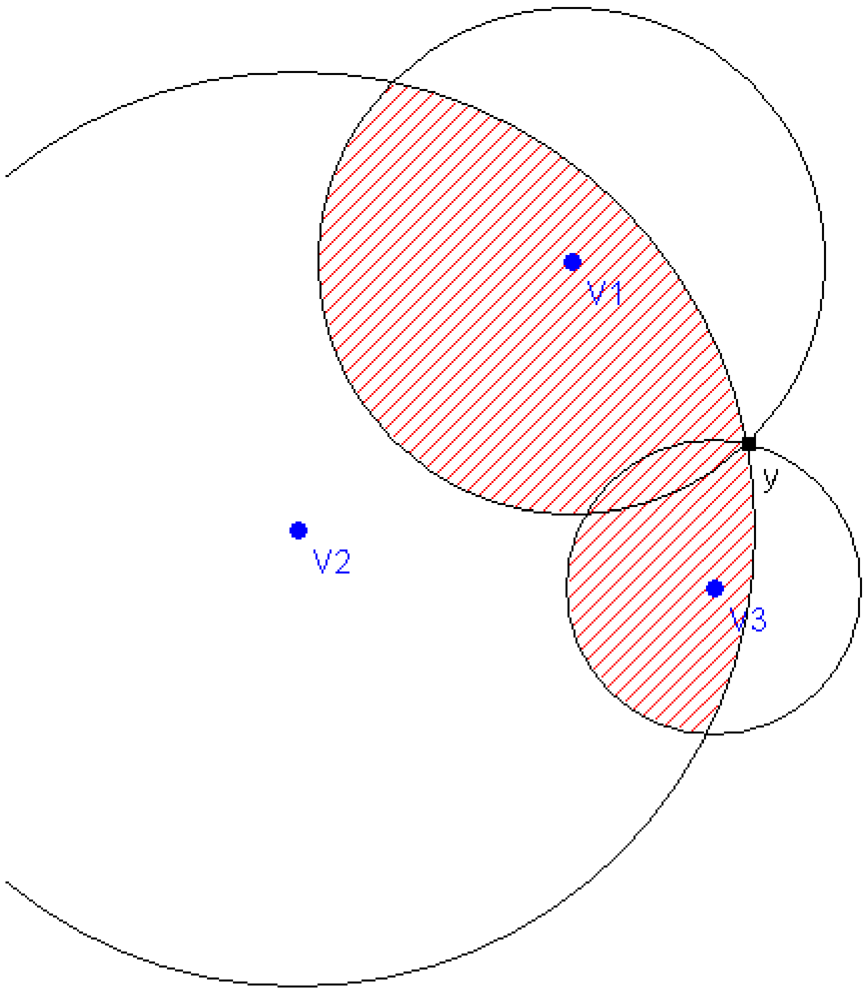


Win set of x



W(x) is the three shaded petals.

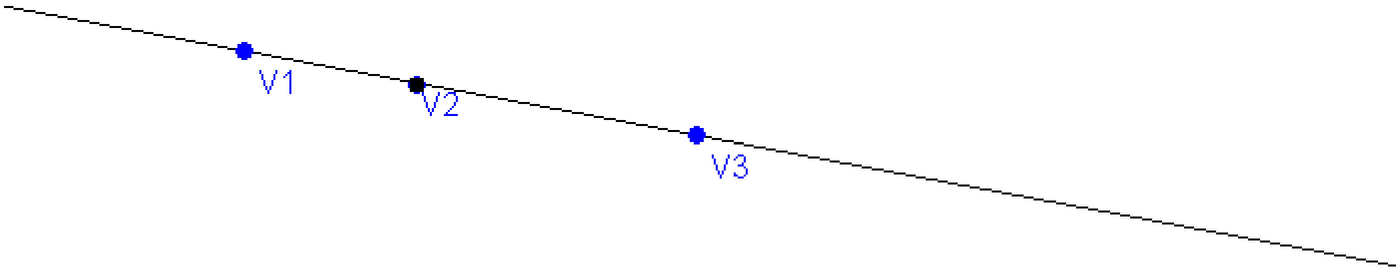
Win set of y



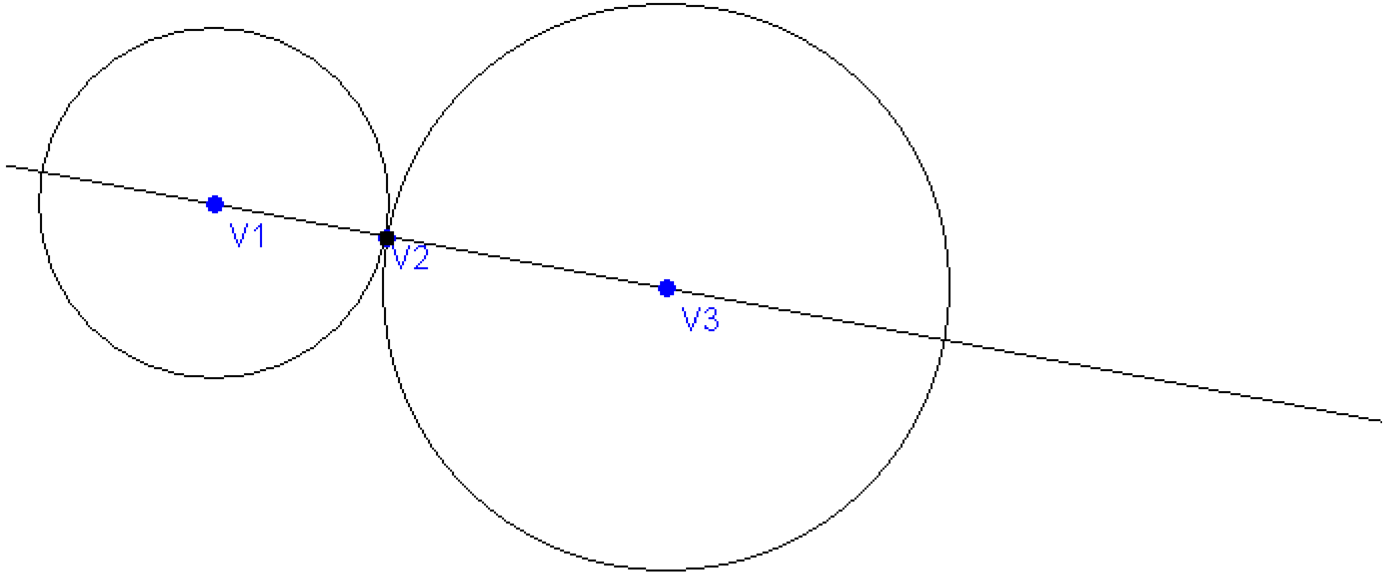
When is the core non-empty in two dimensions?

Put Differently:

- What alternatives are in equilibrium? Under what conditions?
- What alternatives have completely empty win sets?



Core non-empty



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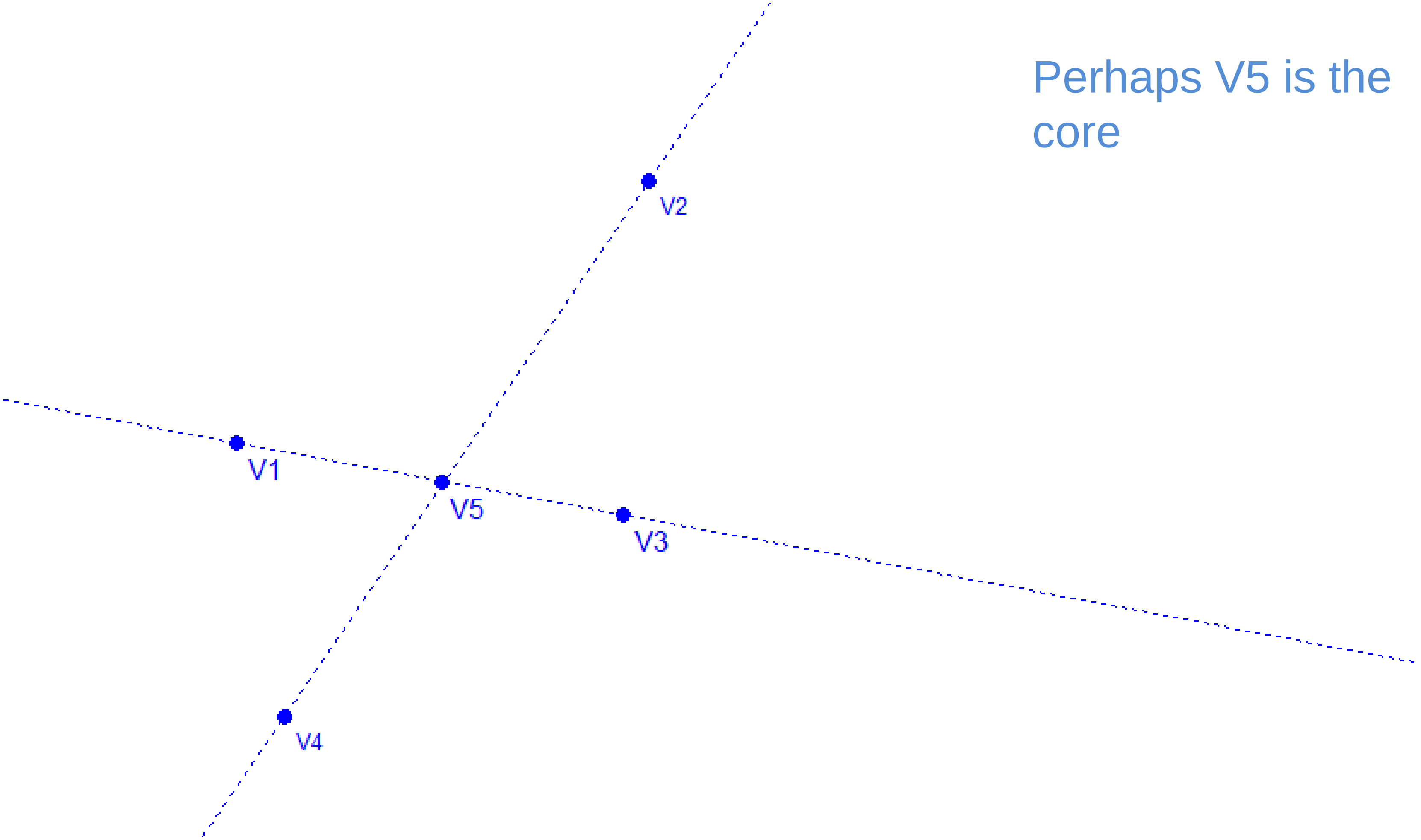
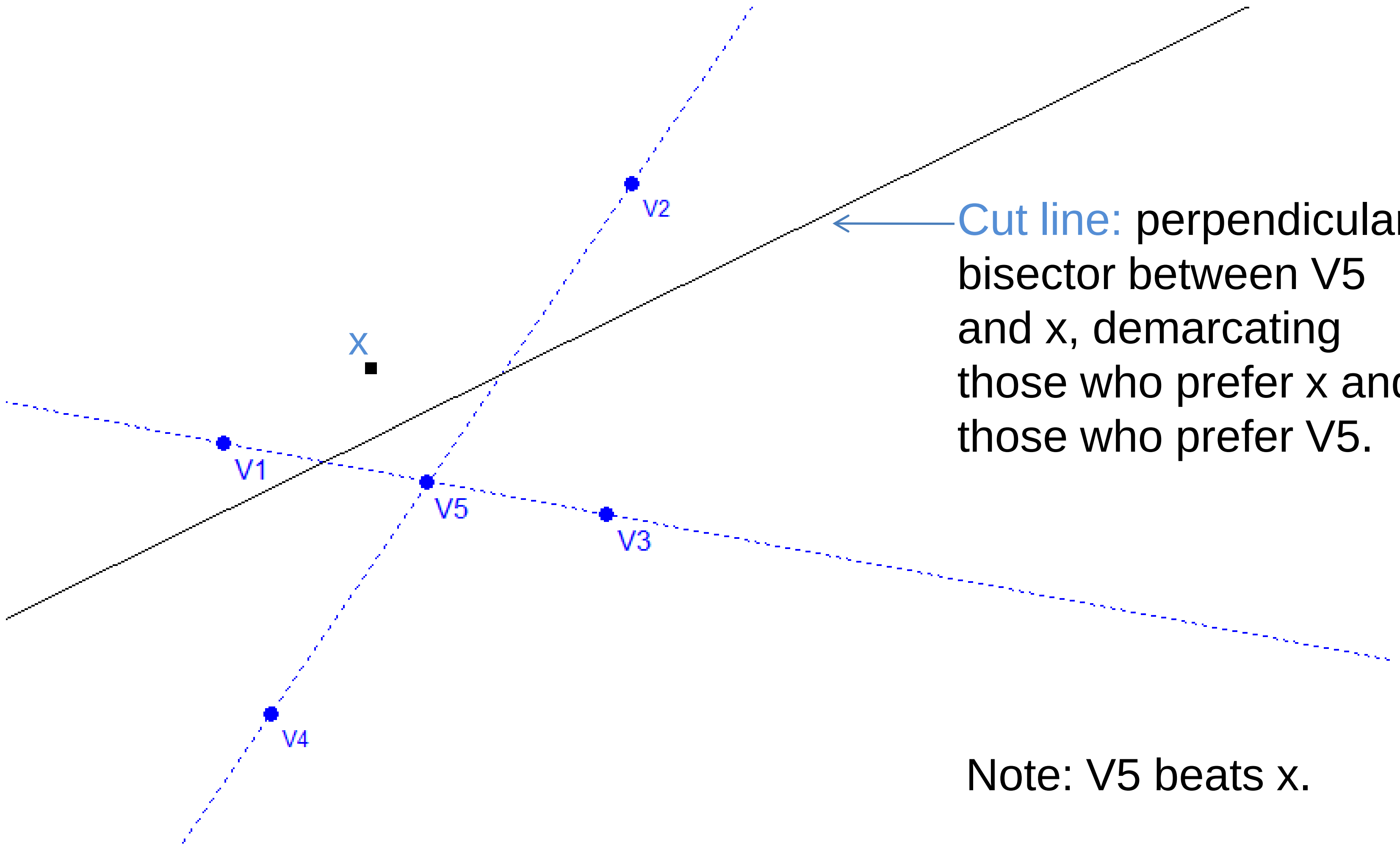


Perhaps V5 is the core

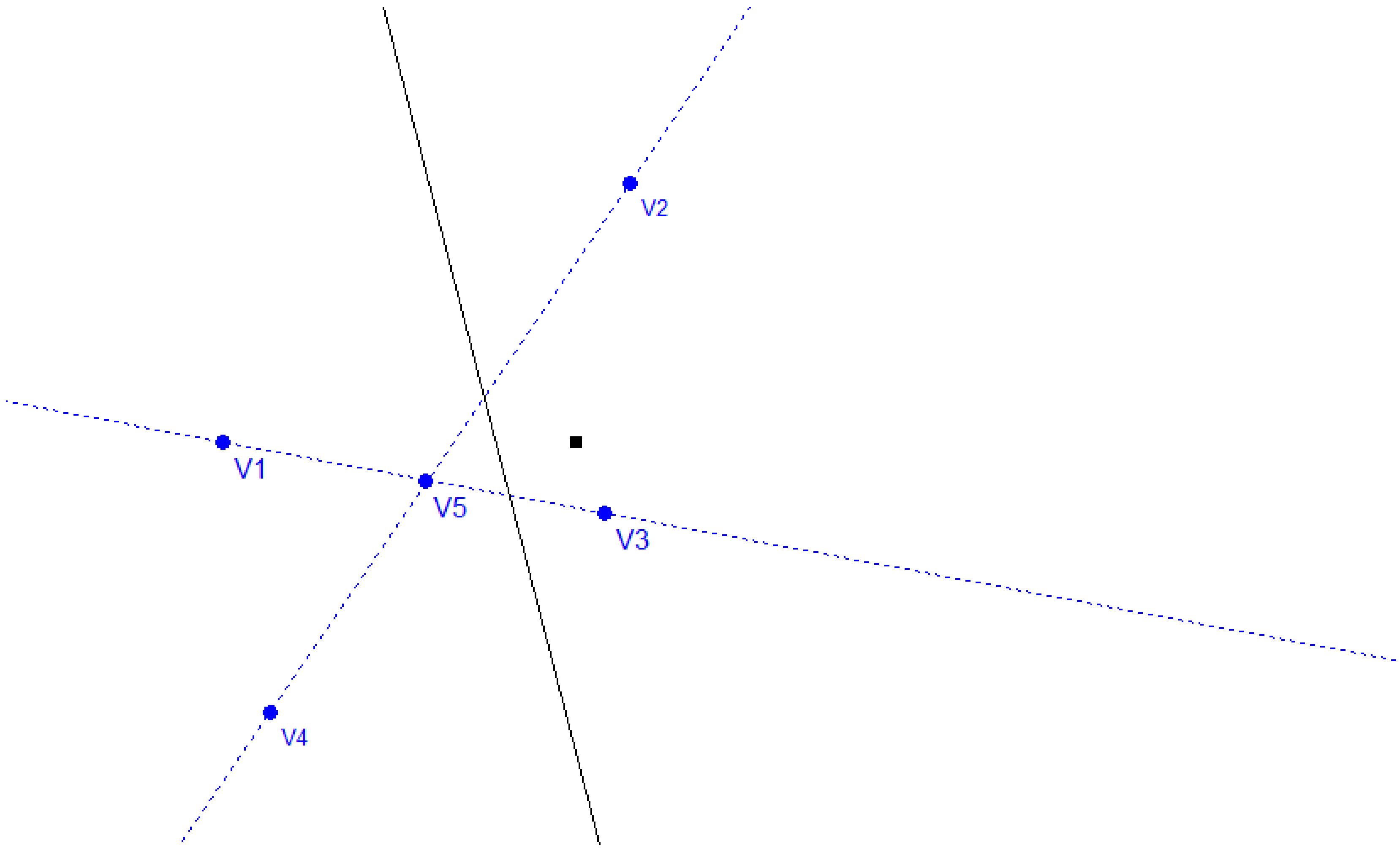
When does the core exist in two dimension?



Cut line: perpendicular bisector between V5 and x, demarcating those who prefer x and those who prefer V5.

Note: V5 beats x.

When is the core non-empty in two dimensions?



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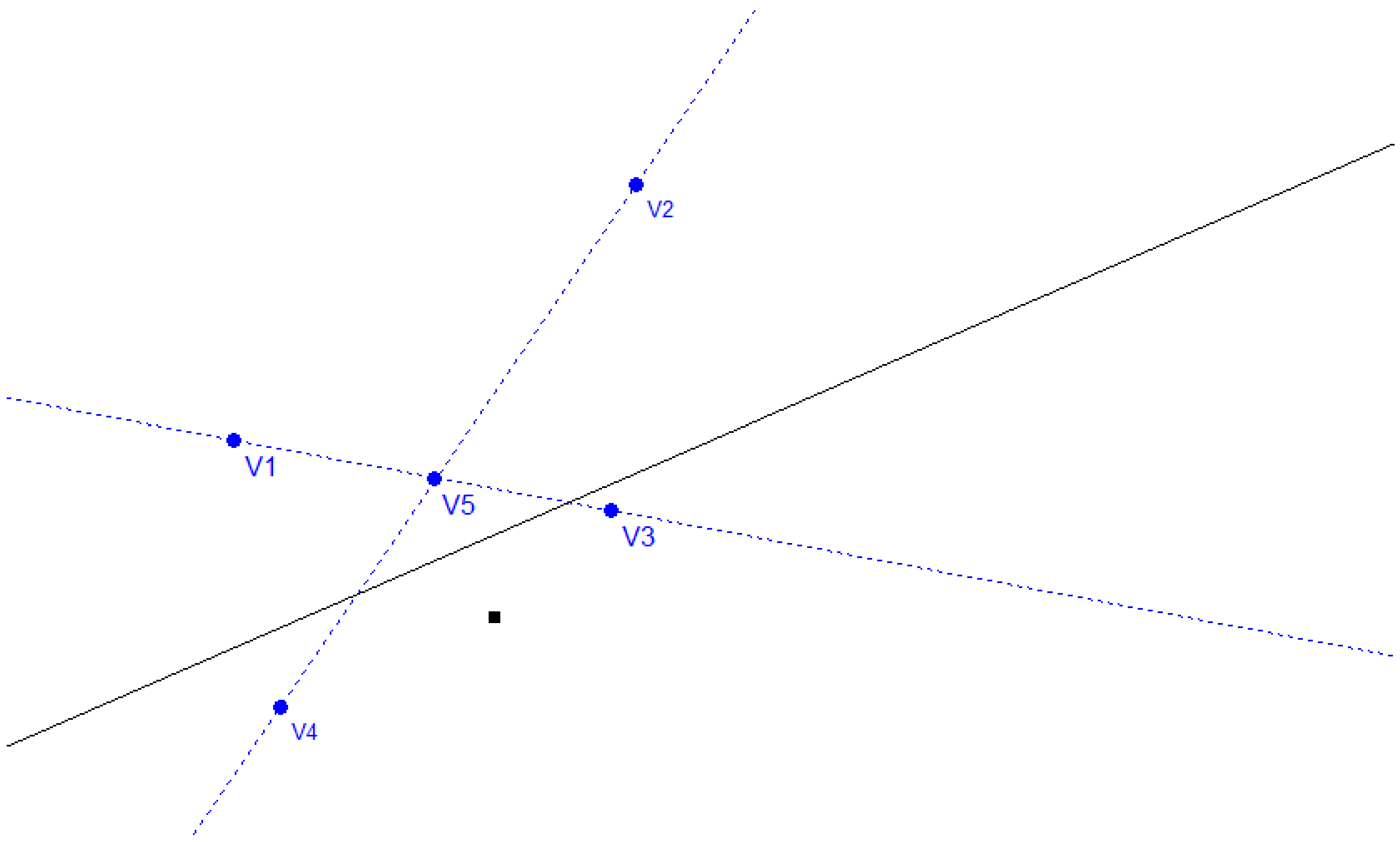

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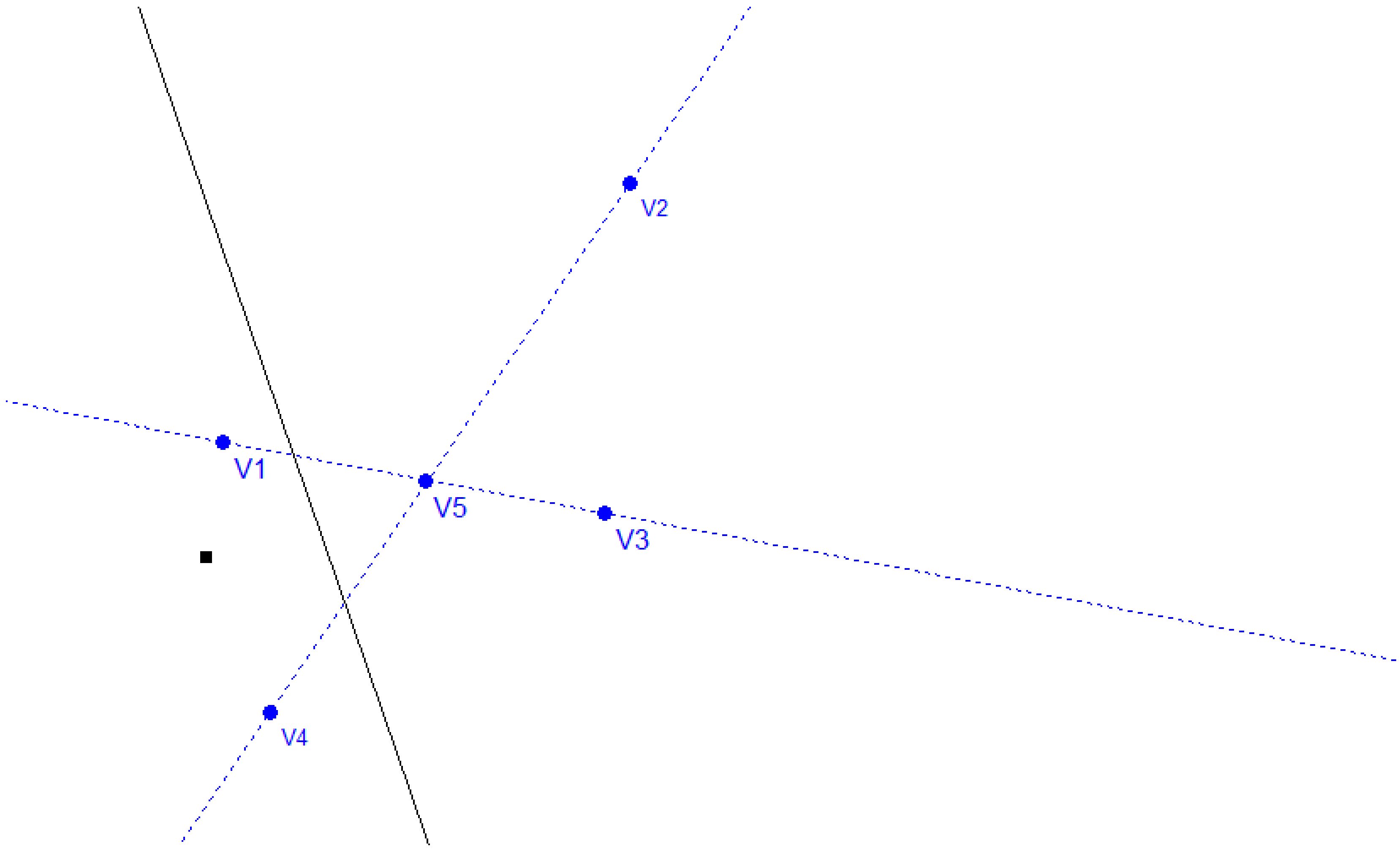
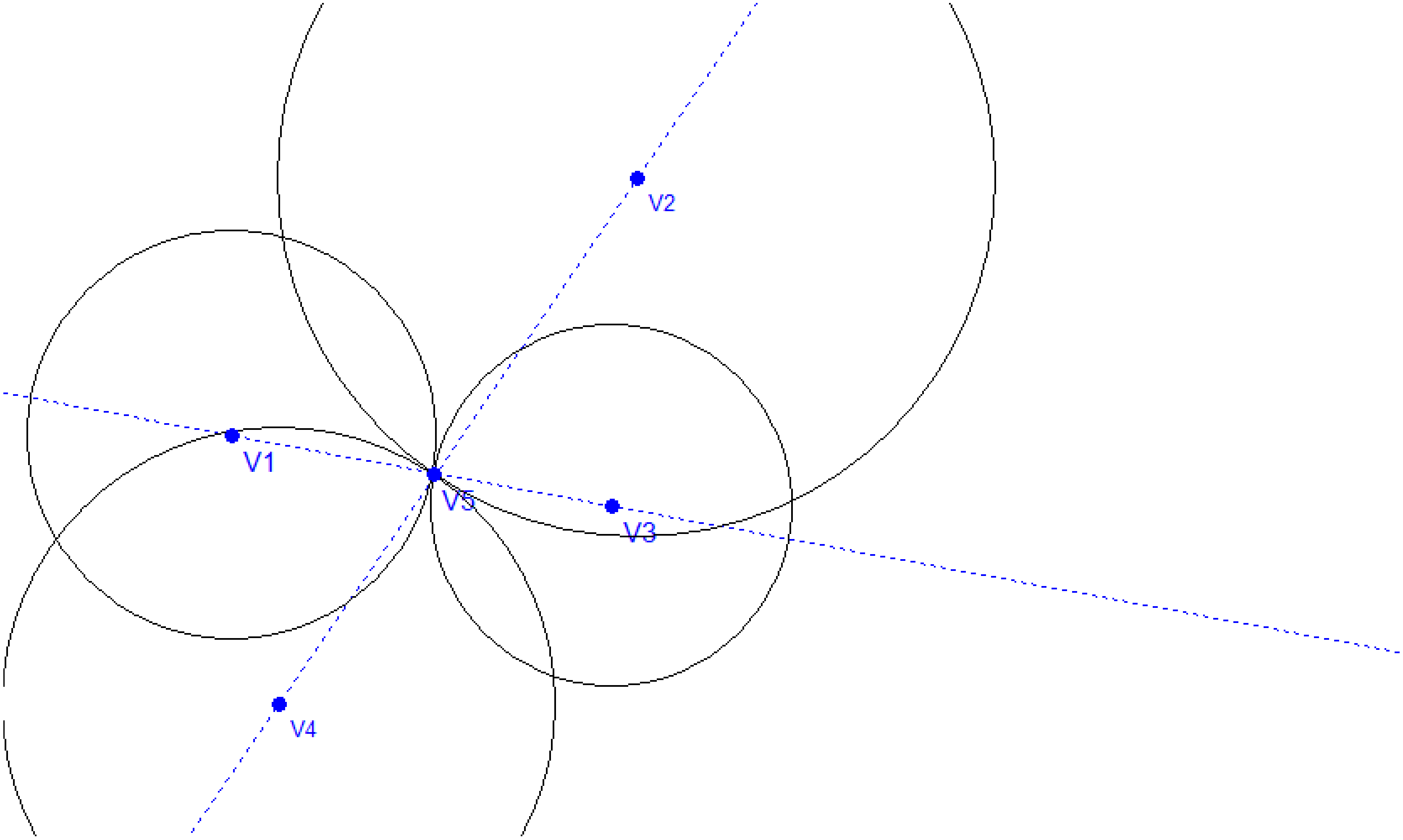


Core is non-empty
(i.e., V5 is an equilibrium)



Core is non-empty
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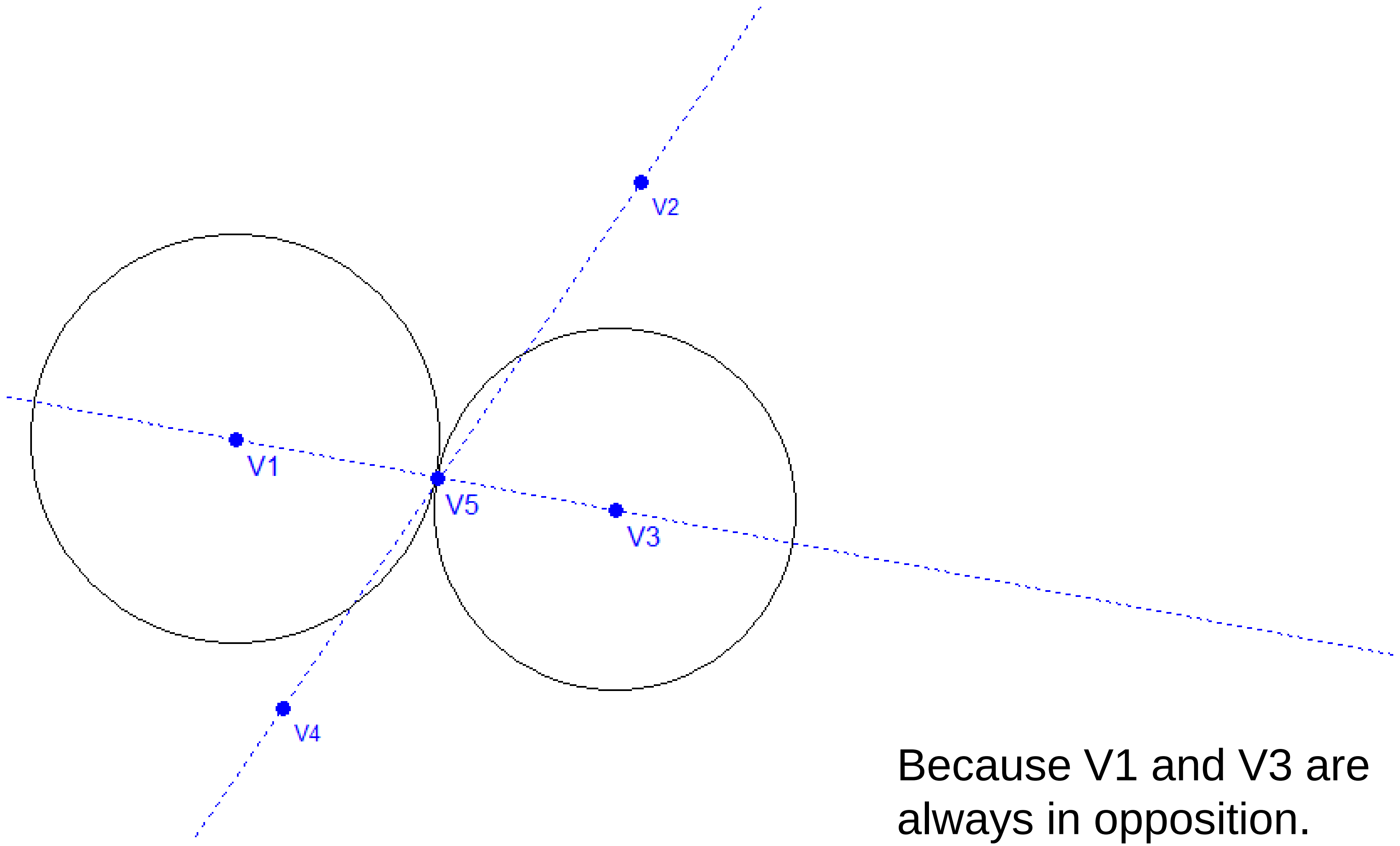

Because V1 and V3 are always in opposition.

Core is non-empty
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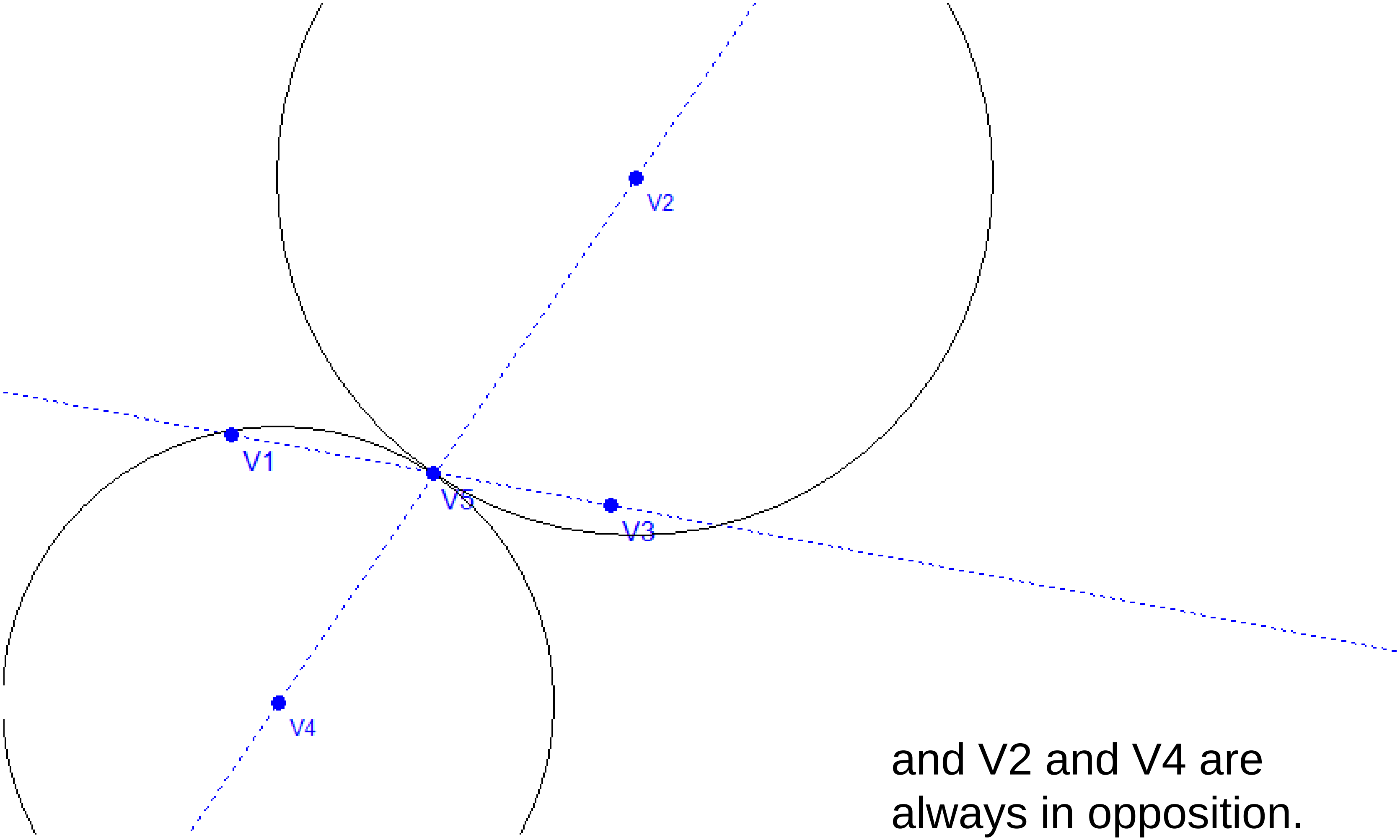
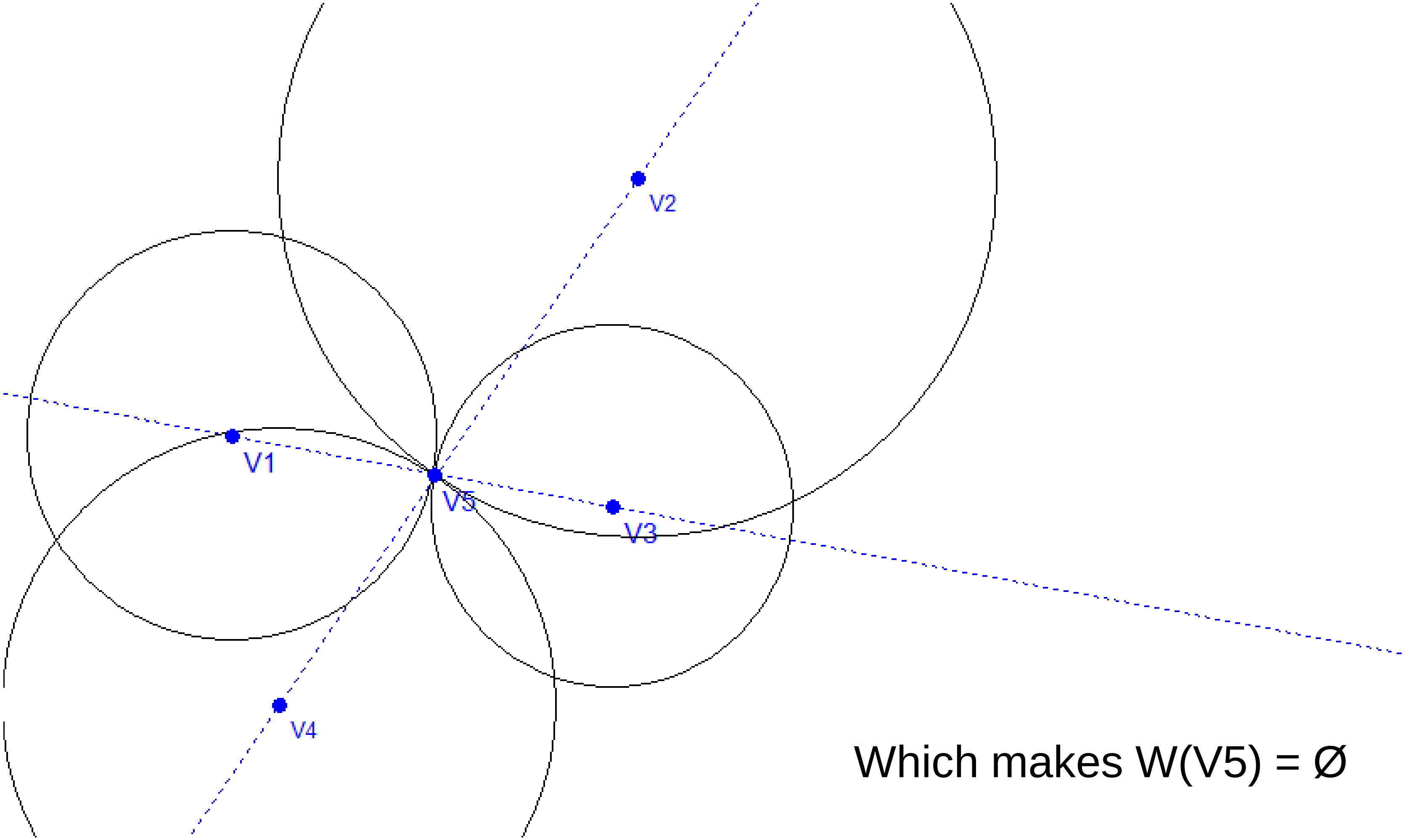


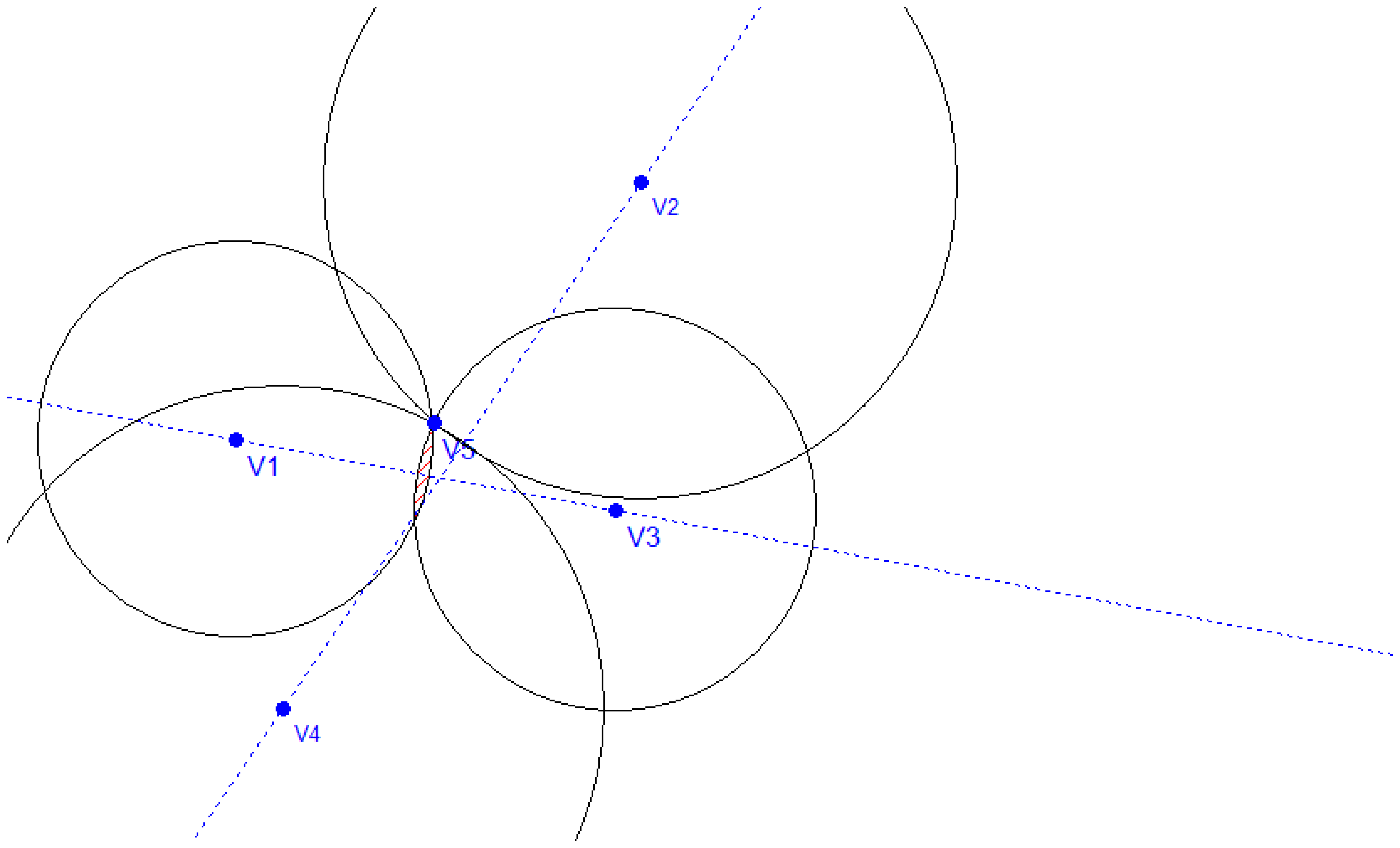
and V2 and V4 are
always in opposition.

Core is non-empty
(i.e., V5 is an equilibrium)

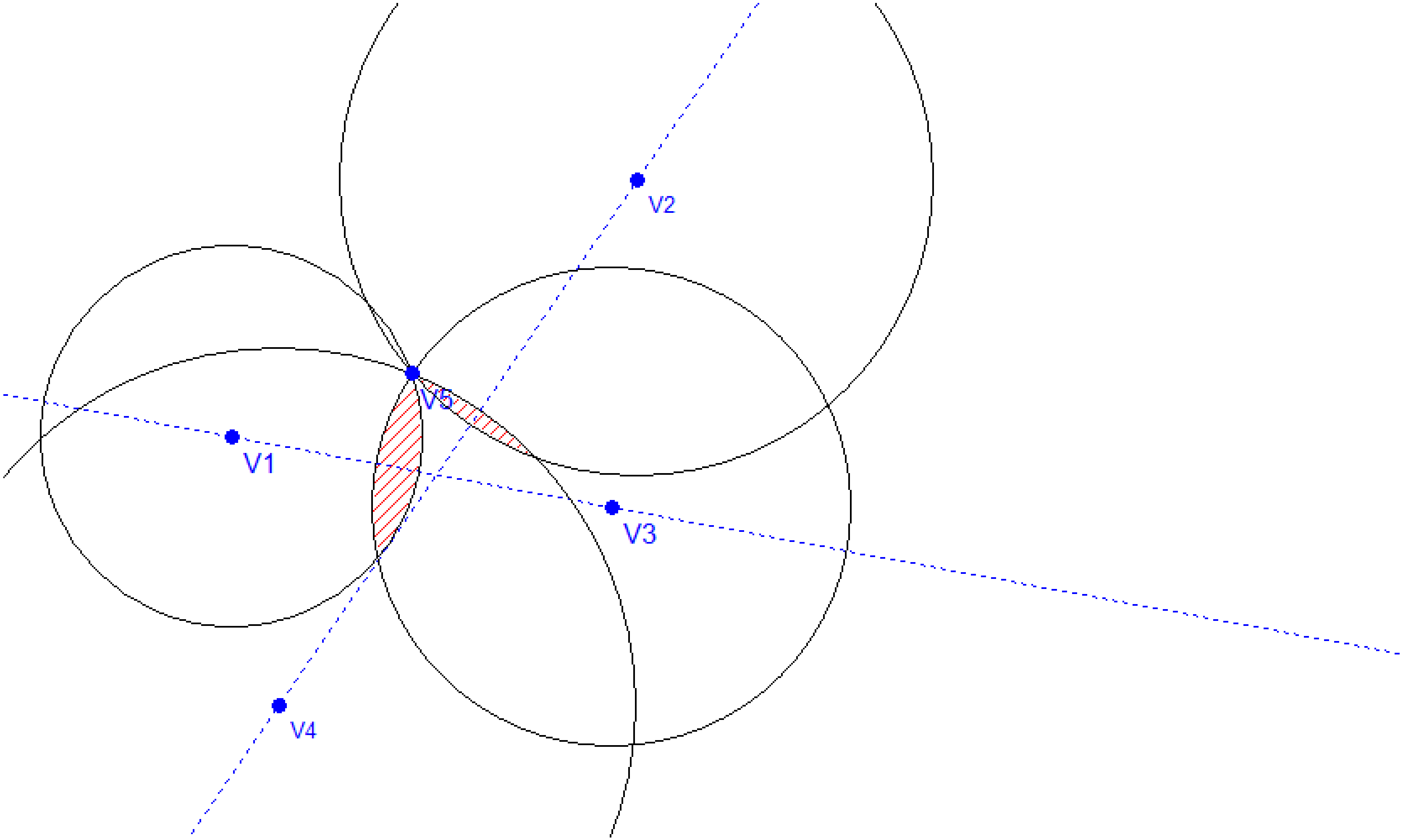


Which makes W(V5) = Ø

Ex 2: Core *is* empty
(i.e., no equilibrium)



Ex 3: Core *is* empty
(i.e., no equilibrium)



Ex 4: Core *is* empty
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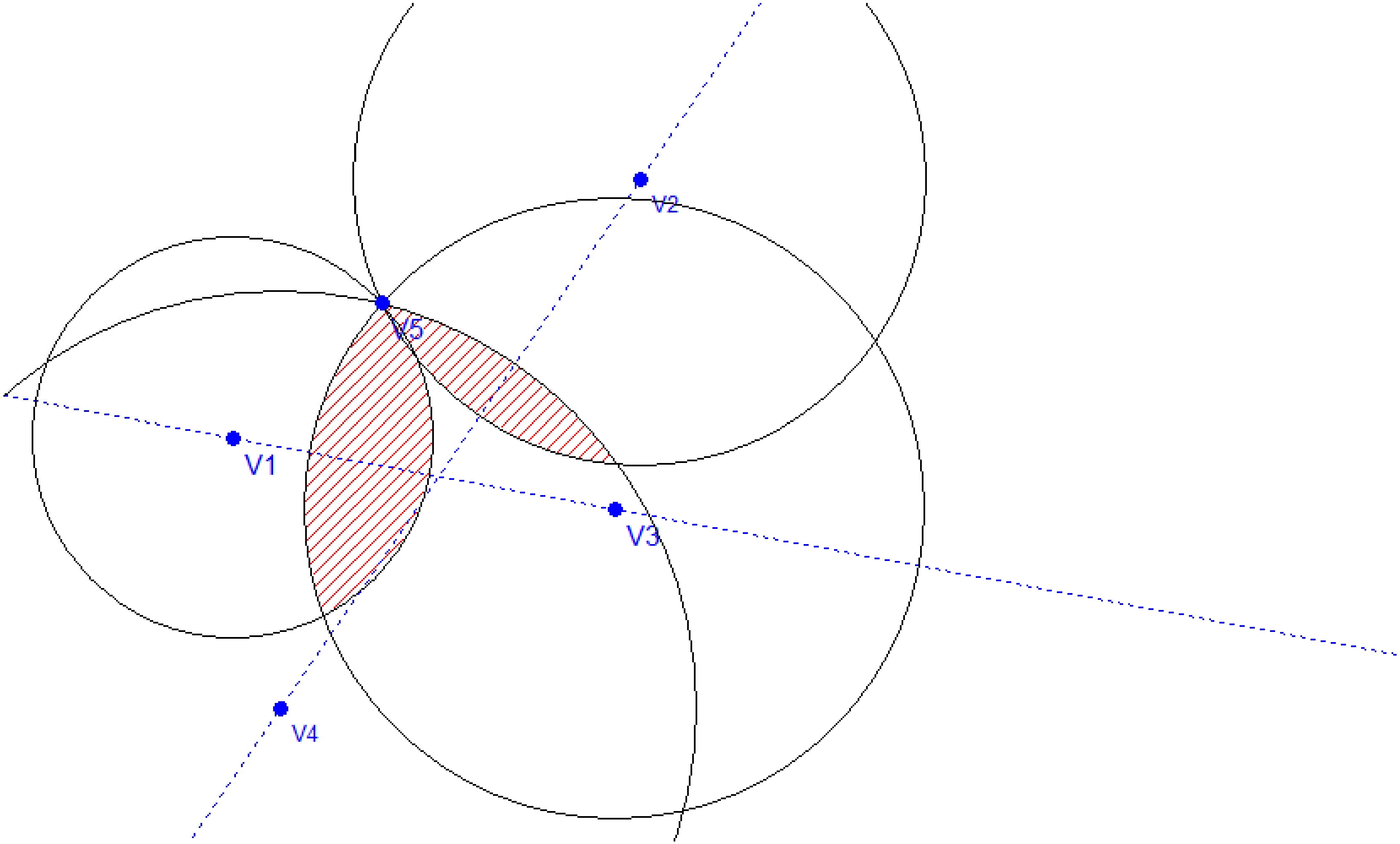

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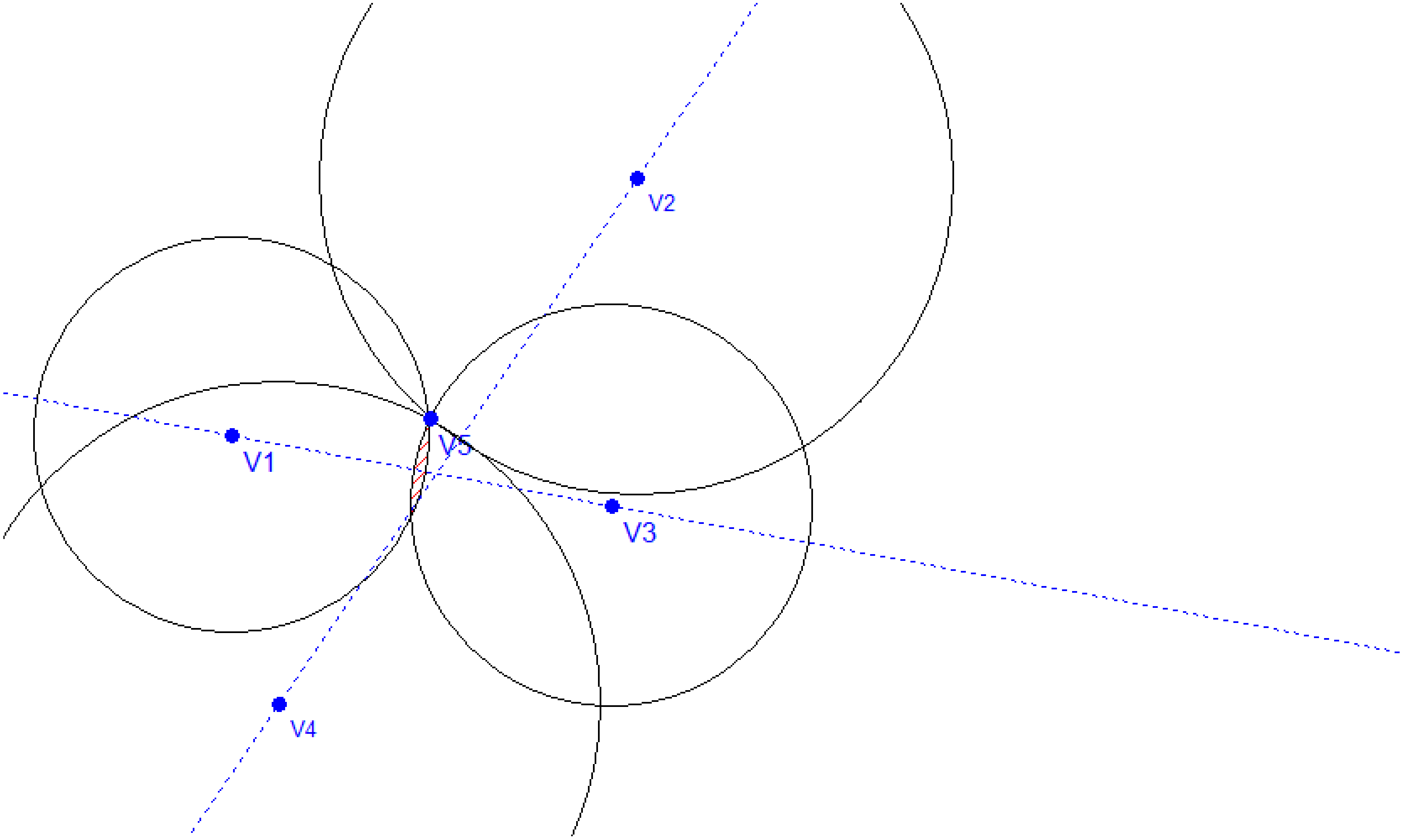

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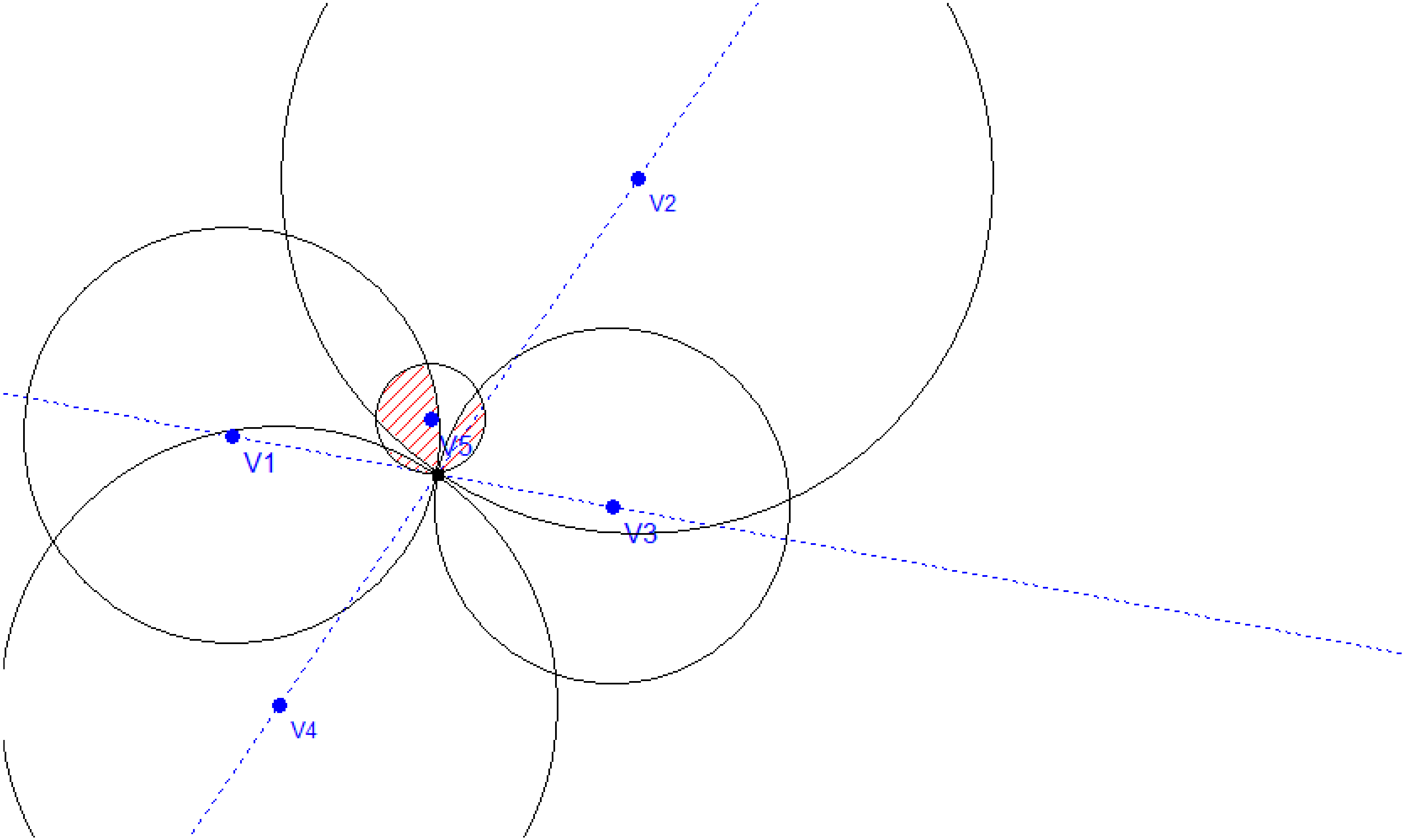
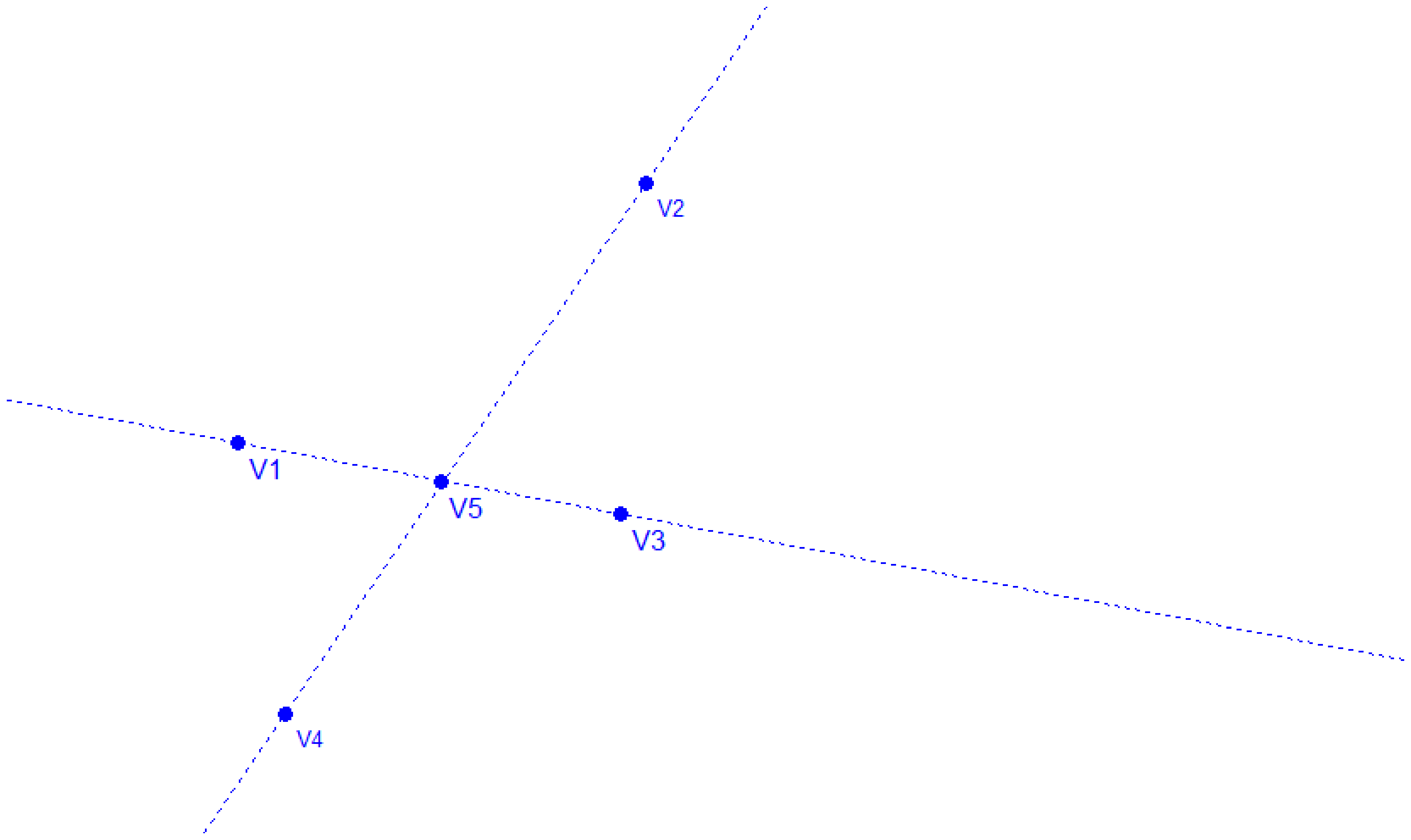


Plott conditions

- The **Plott conditions** are satisfied if and only if ideal points are distributed in a radialy symmetric fashion around a policy x* and x* is a voter's ideal point
- **Radial symmetry** means that ideal points on opposite sides of x* can be paired by a line through x* and the lines defined by each pair intersect at x*
- The ideal point x* is sometimes called the **generalized median voter**.

Proposition In the spatial model with Euclidean preferences and an odd number of voters, the majority rule core is x* if and only if the Plott conditions hold

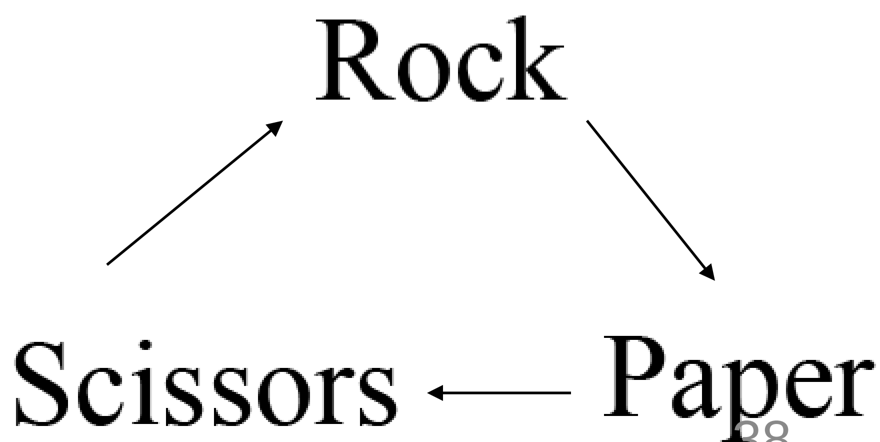
Plott conditions



Multidimensional Spatial Voting

Vote Cycle

1. Intransitive group preferences caused by the voting rule.
2. Intransitivity Example: Rock, Scissors, Paper.
3. Note: the core would be empty if the alternatives were in a vote cycle.



Multidimensional Spatial Voting

Vote Cycle

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1	2	3
X	Z	Y
Y	X	Z
Z	Y	X

X vs Z:

Multidimensional Spatial Voting

Vote Cycle

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1	2	3
X	Z	Y
Y	X	Z
Z	Y	X

X vs Z: Z wins (2 to 1)

Multidimensional Spatial Voting

Vote Cycle

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1	2	3
X	Z	Y
Y	X	Z
Z	Y	X

X vs Z: Z wins (2 to 1)

Z vs Y: Y wins (2 to 1)

Multidimensional Spatial Voting

Vote Cycle

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2. Intransitivity Example: Rock, Scissors, Paper.
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1	2	3
X	Z	Y
Y	X	Z
Z	Y	X

X vs Z: Z wins (2 to 1)

Z vs Y: Y wins (2 to 1)

Y vs X: X wins (2 to 1)

Multidimensional Spatial Voting

Vote Cycle

1. Intransitive group preferences caused by the voting rule.
2. Intransitivity Example: Rock, Scissors, Paper.
3. Note: the core would be empty if the alternatives were in a vote cycle.

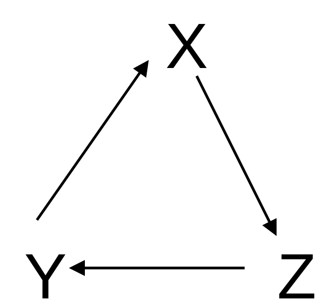
So we have demonstrated the possibility of majority rule vote cycles, but how general is the problem?

1	2	3
X	Z	Y
Y	X	Z
Z	Y	X

X vs Z: Z wins (2 to 1)

Z vs Y: Y wins (2 to 1)

Y vs X: X wins (2 to 1)



Intransitivity

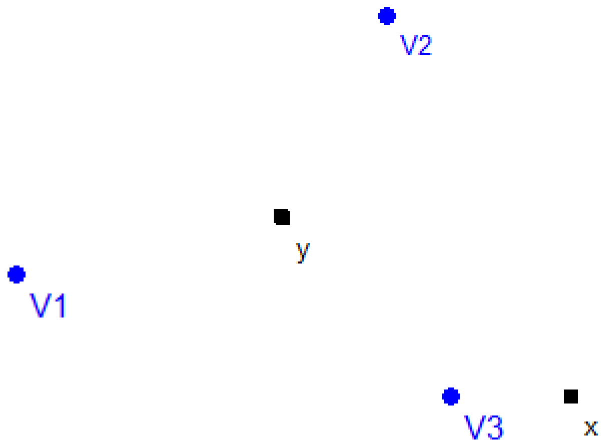
McKelvey's Theorem

In the spatial model with Euclidean preferences, either the majority rule *core is non-empty* (i.e. something is in equilibrium) or *the entire set of alternatives is in a voting cycle.*

Corollaries

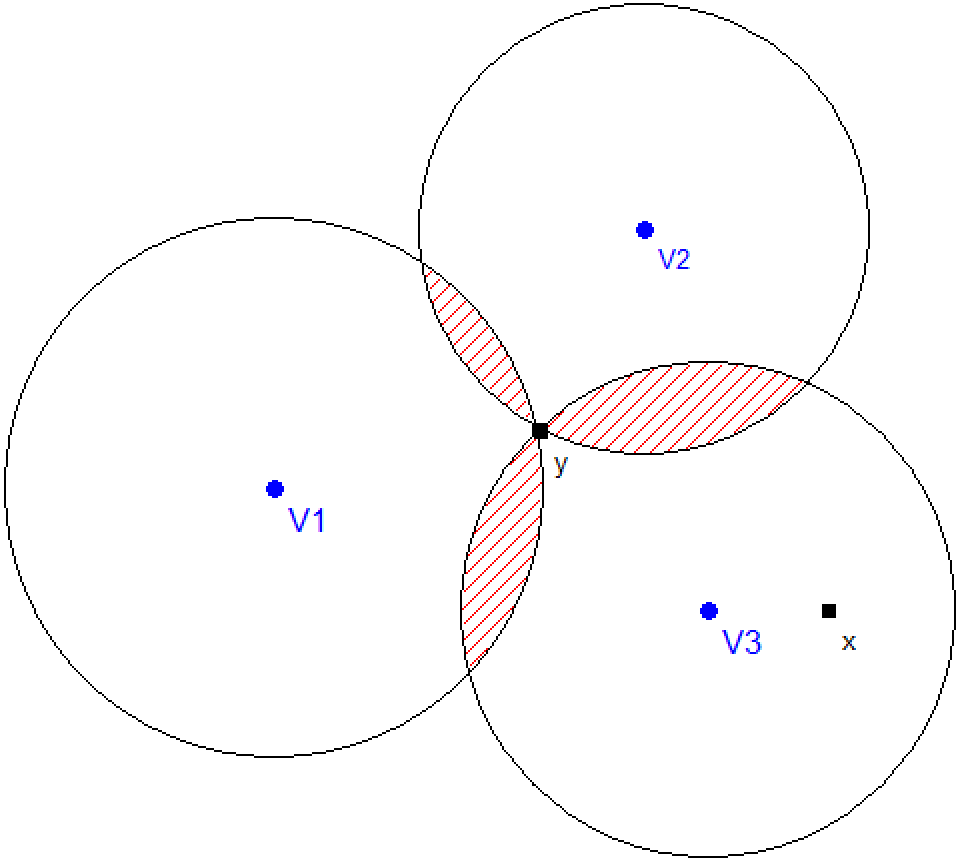
- If the Plott conditions do not hold, then the entire set of alternatives is in a voting cycle.
- If the Plott conditions do not hold, then the majority rule core is empty (i.e. no point is in equilibrium).

Constructing a cycle



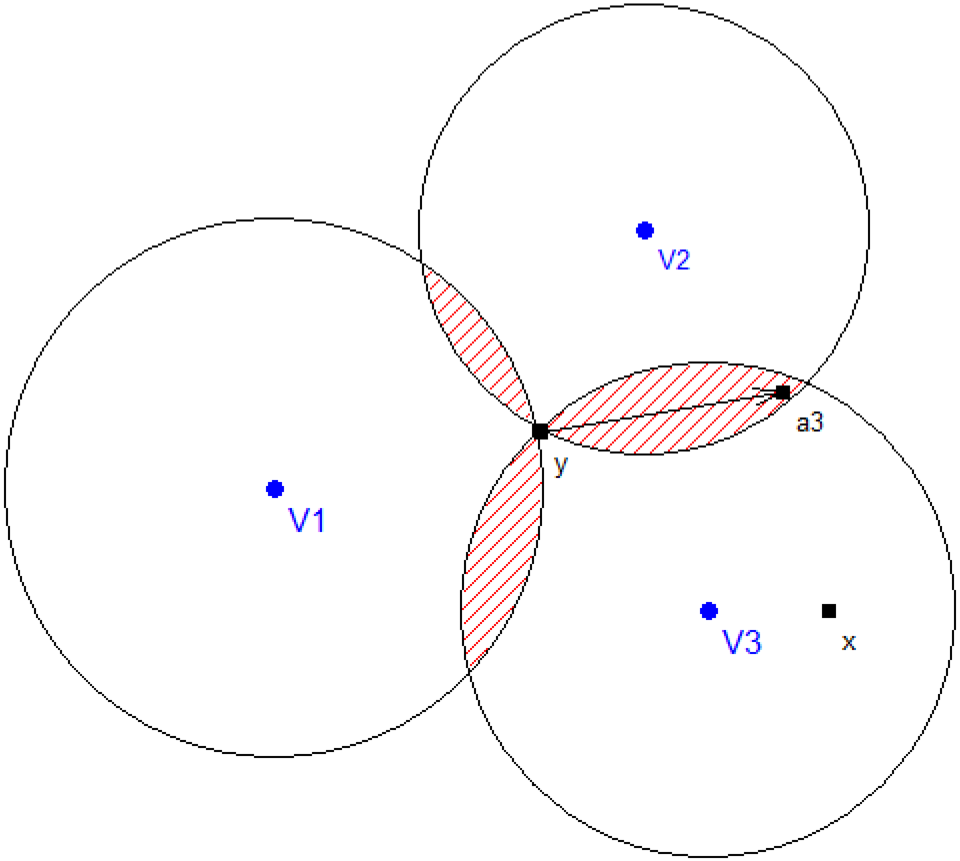
We are going to construct an agenda starting with y and ending in x.

Constructing a cycle

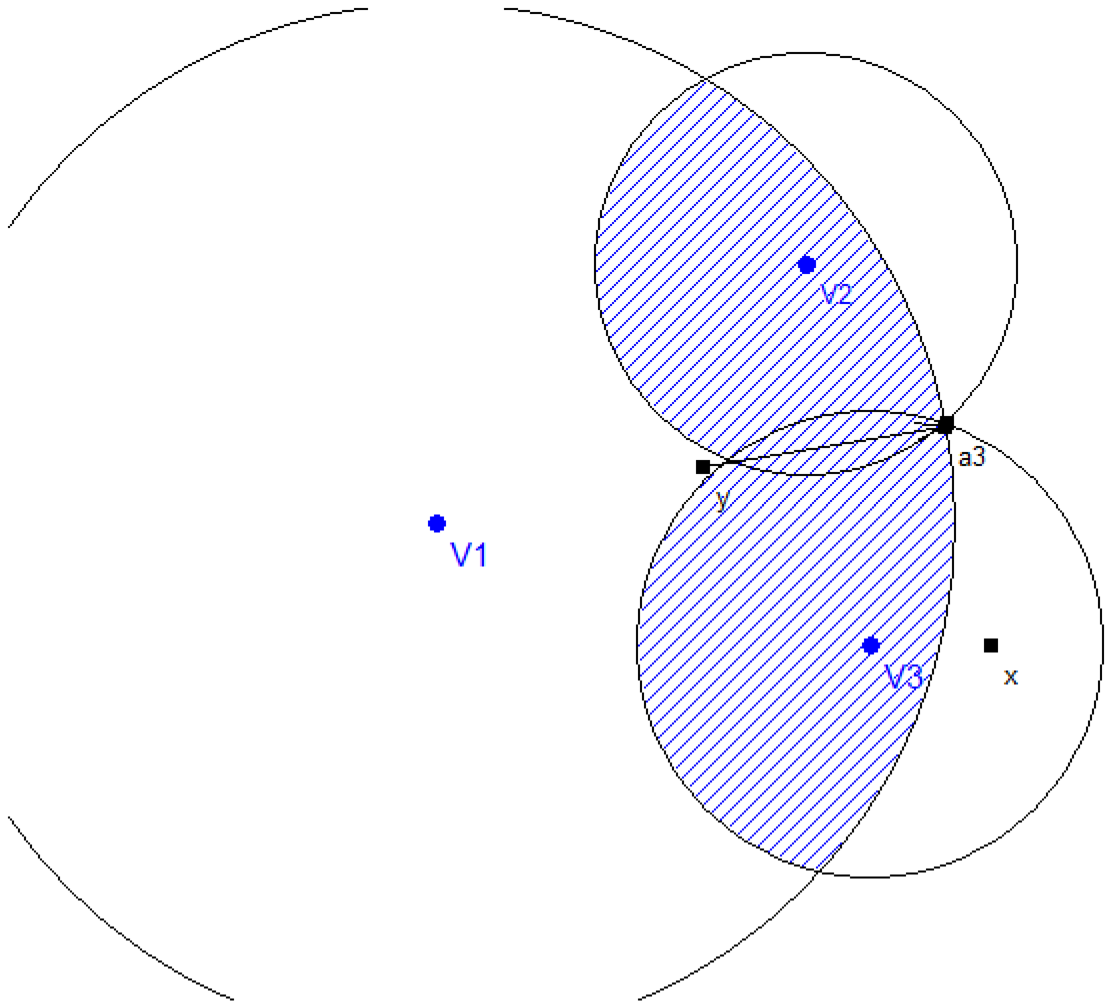


Note that $x \notin W(y)$

Constructing a cycle



Constructing a cycle



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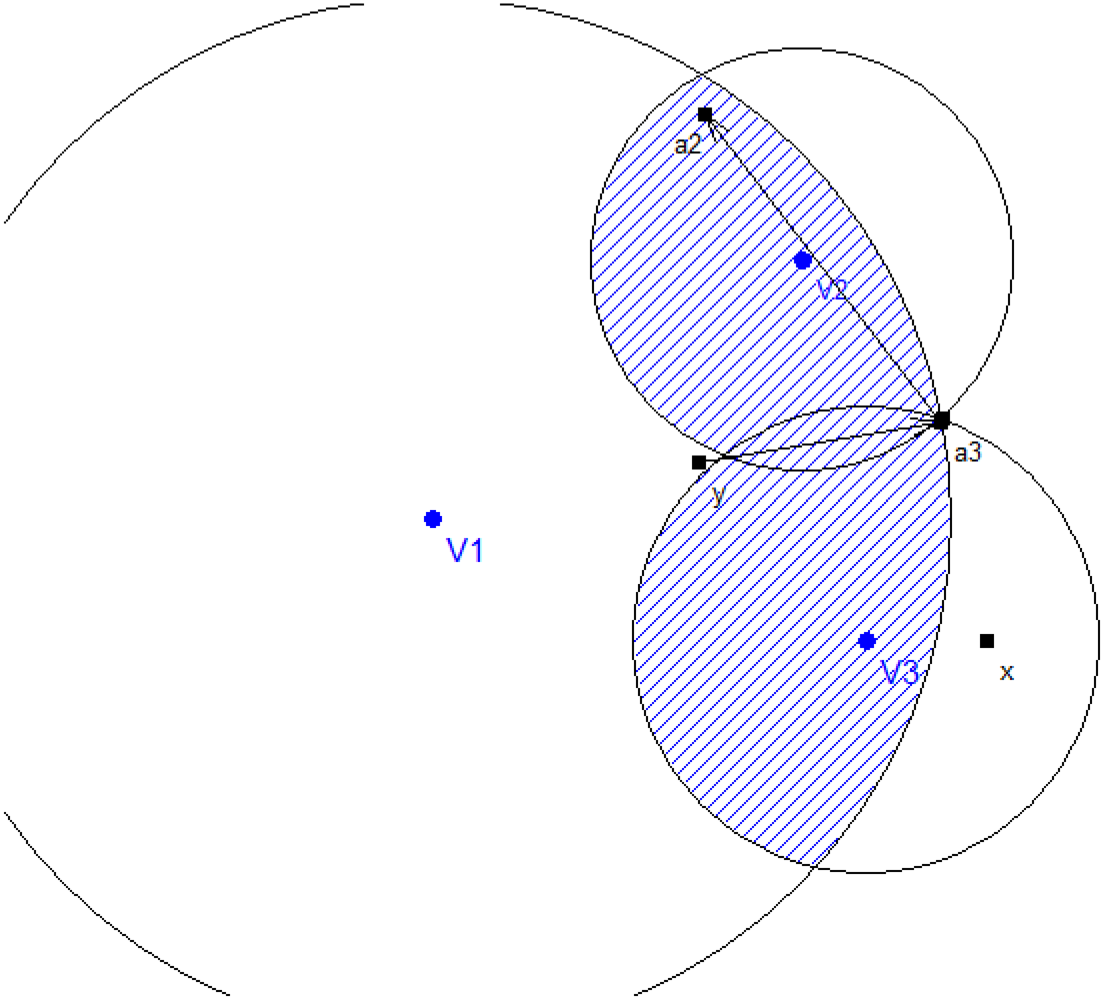
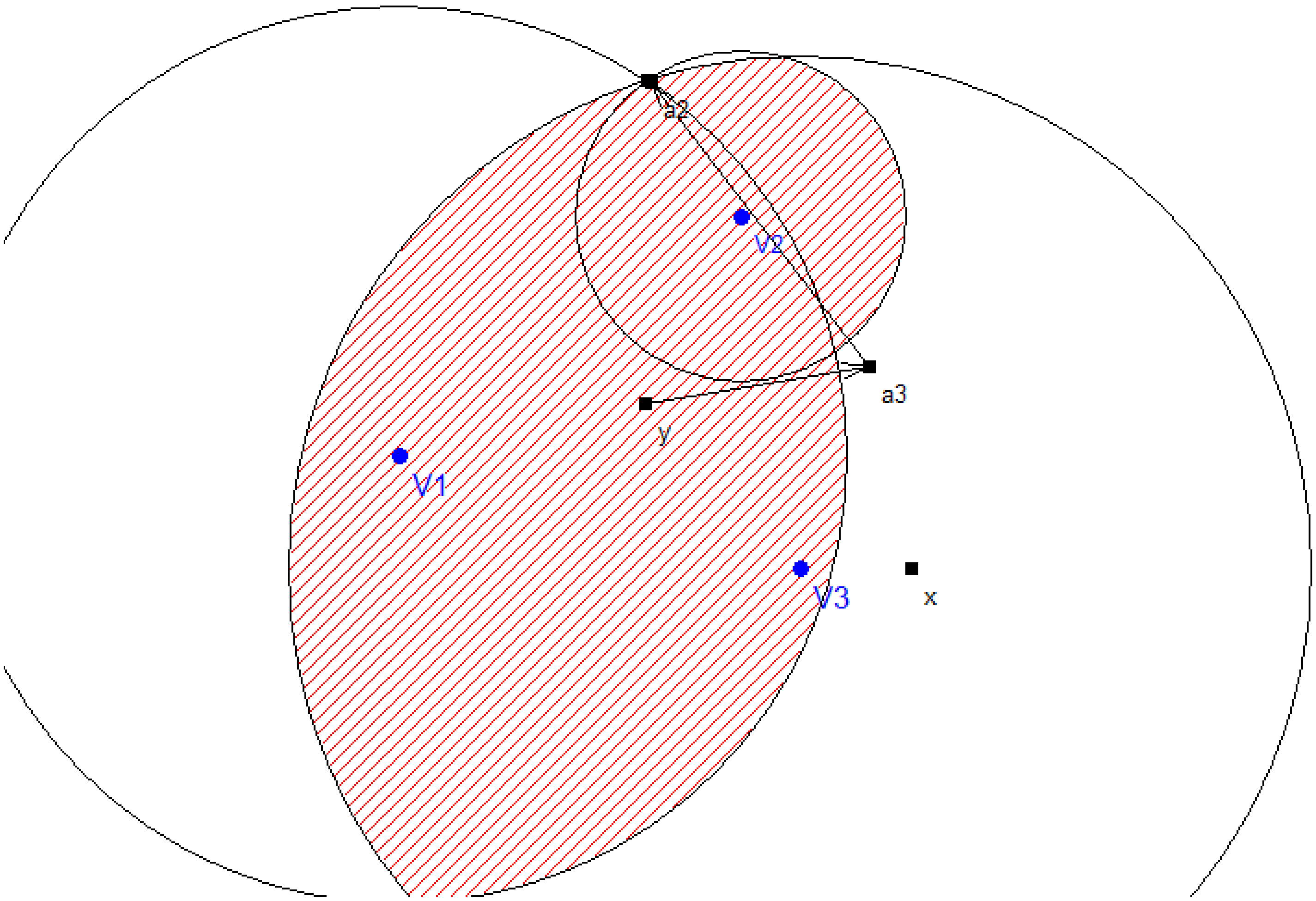


Constructing a cycle



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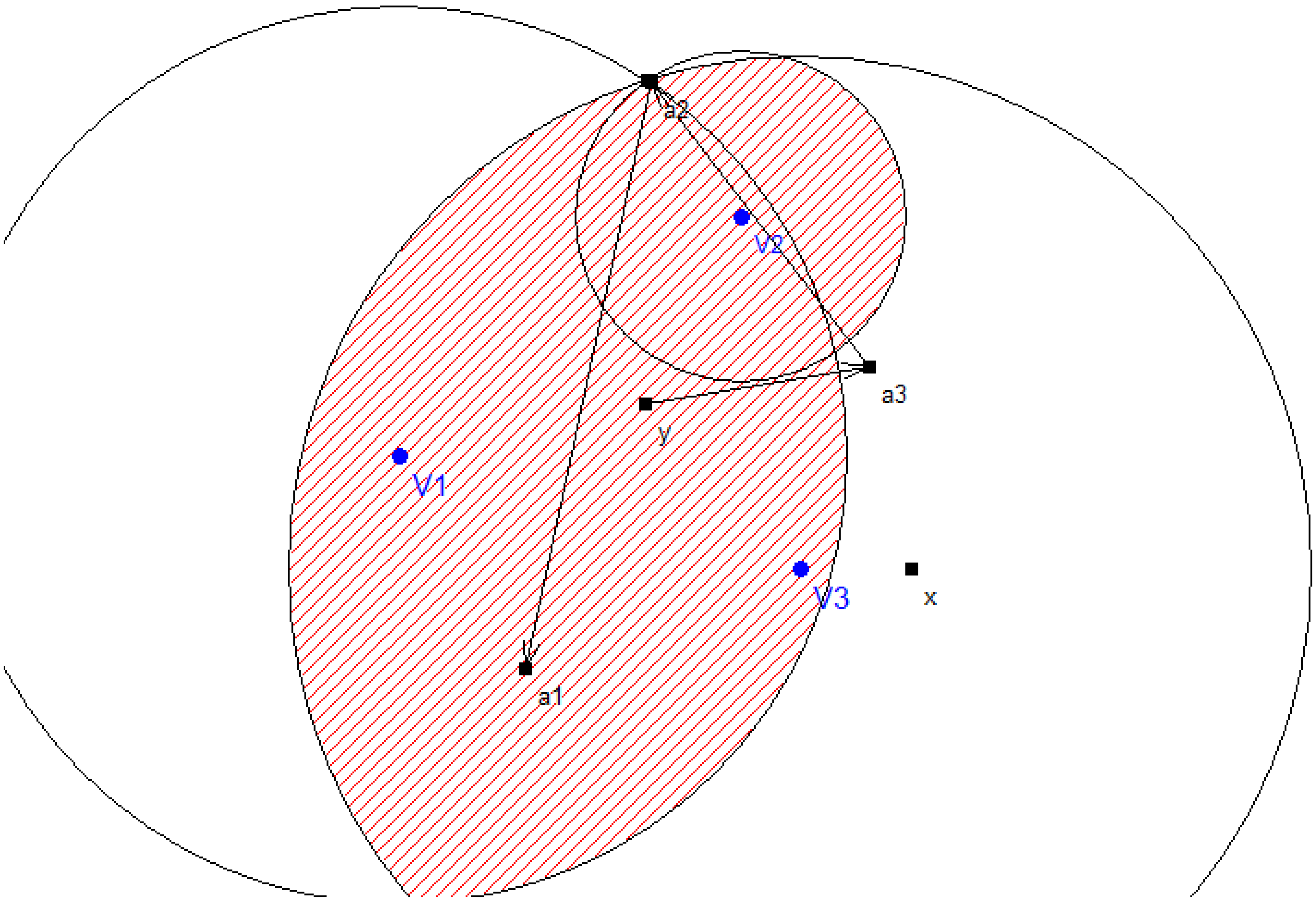
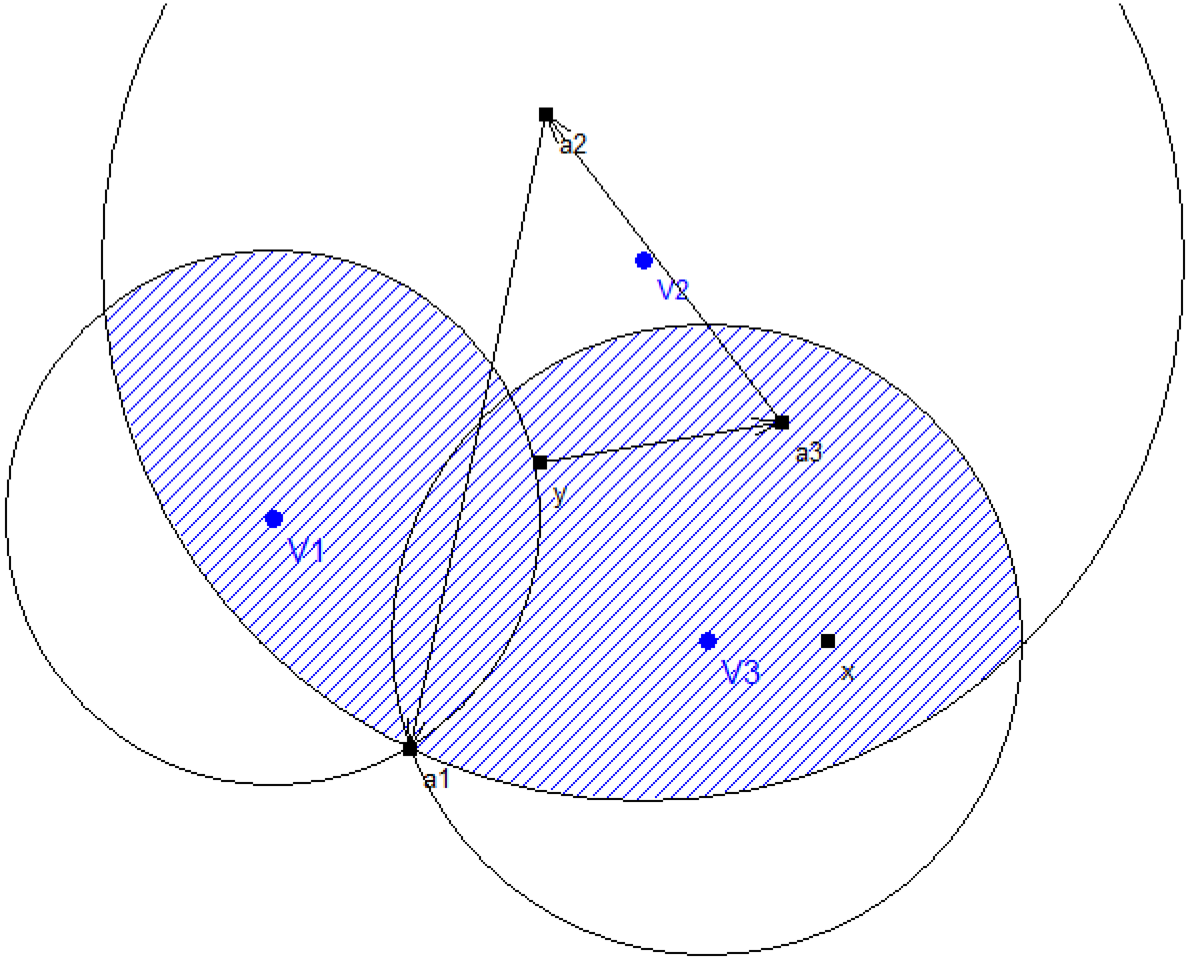
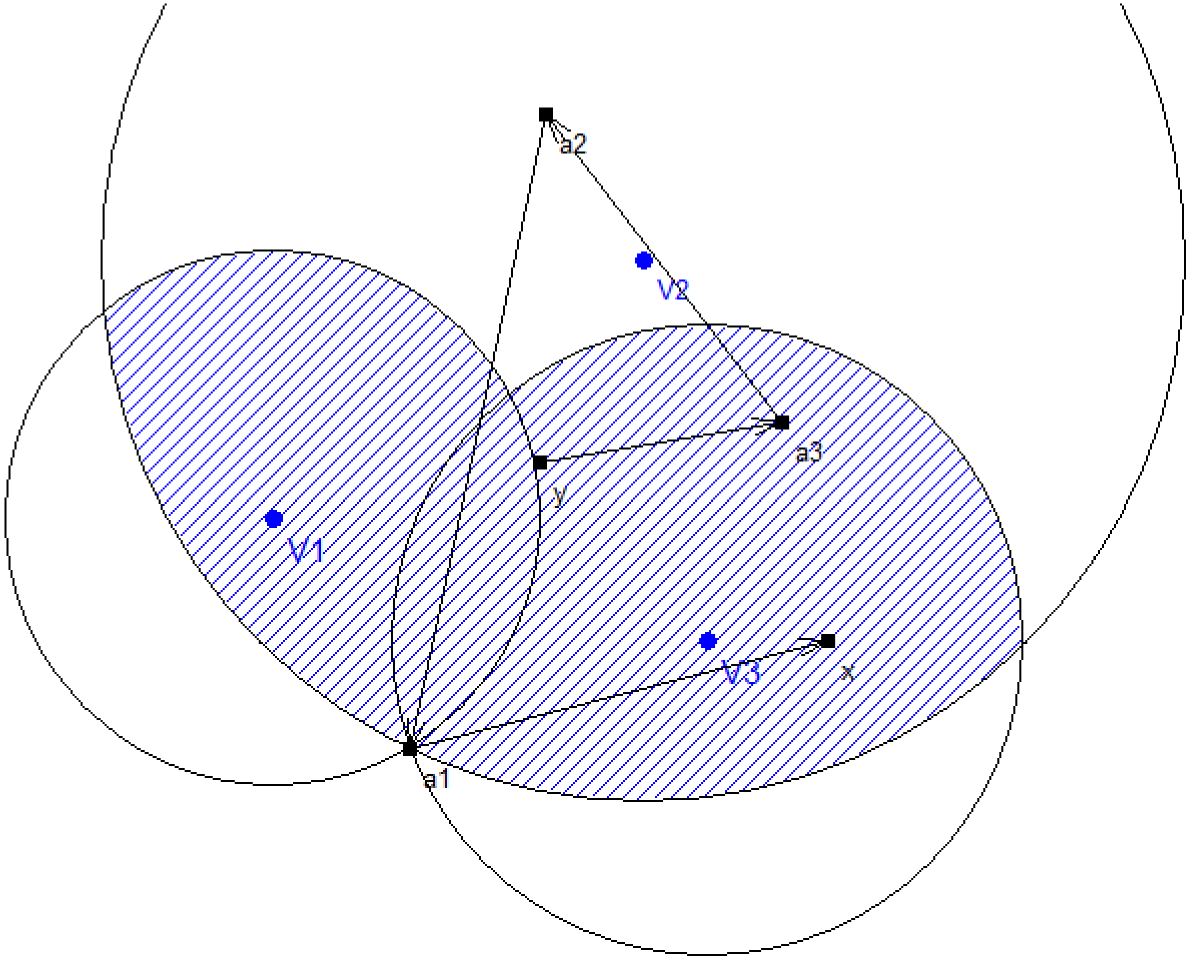


Constructing a cycle



Constructing a cycle



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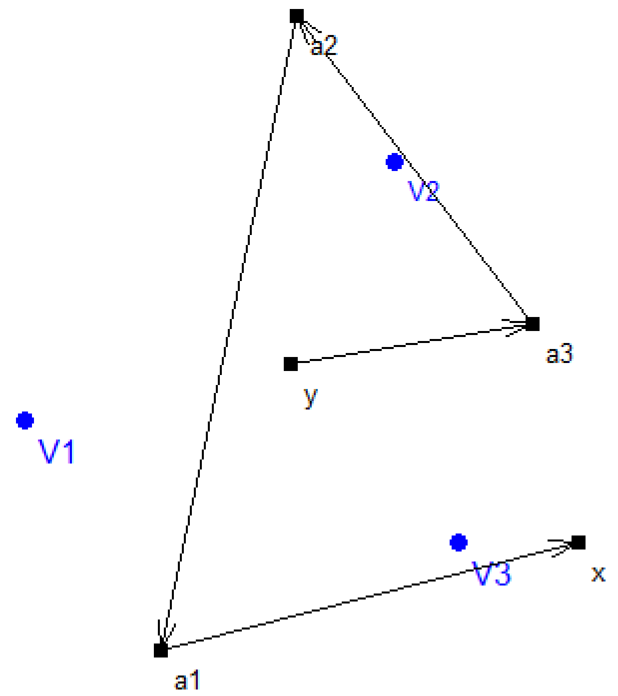
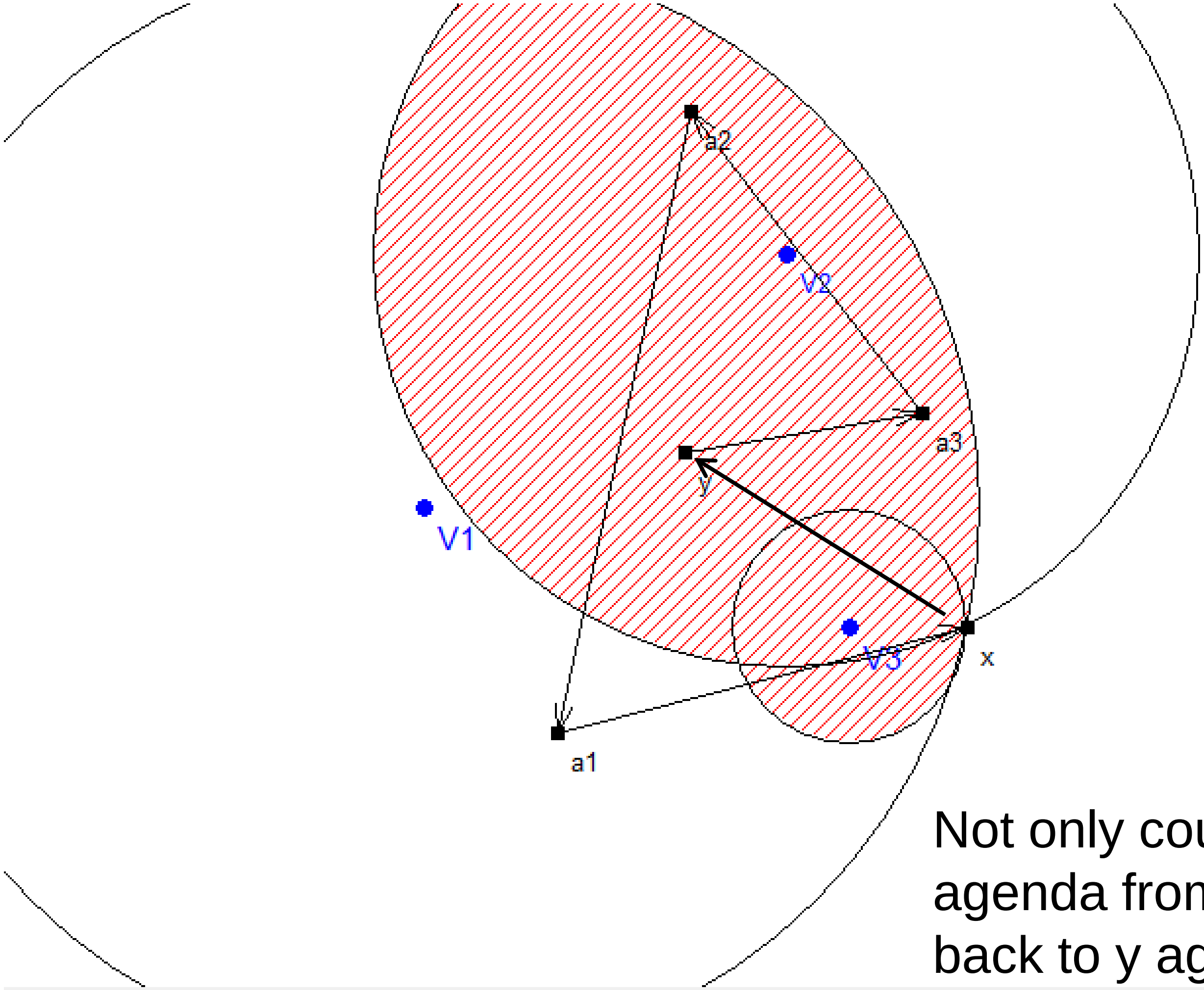


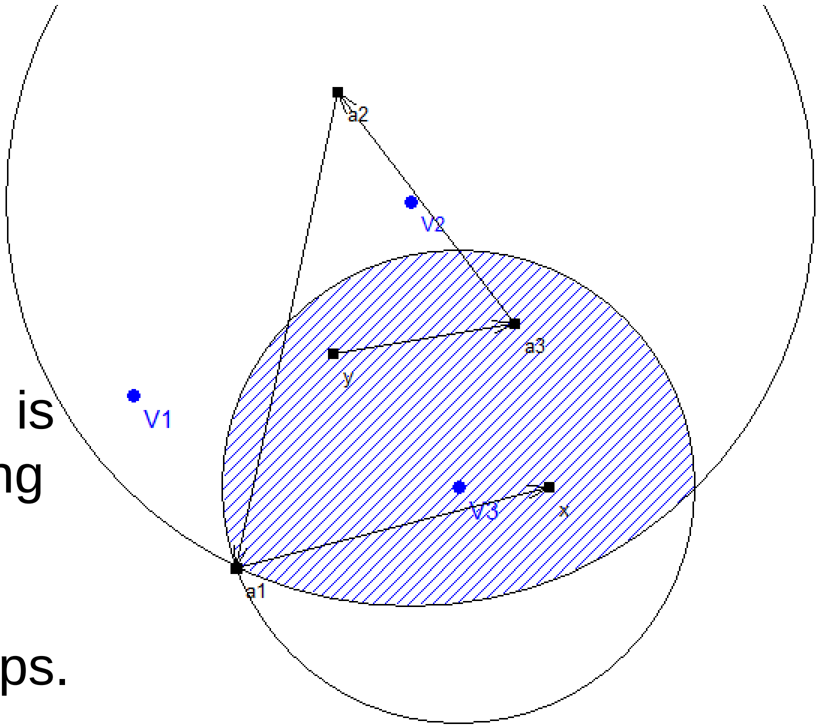
Constructing a cycle



Not only could we construct an agenda from y to x, we can get back to y again (i.e. intransitivity).

Intuition: Shifting majorities

Step	1	2	3	Coalition
a_3Py				
a_2Pa_3				
a_1Pa_2				
xPa_1				

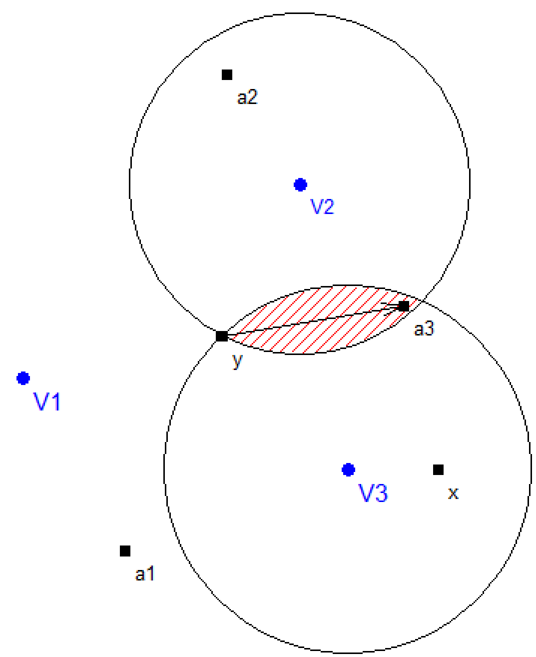


Note that the cycle is explained by shifting coalitions.

Let's follow the steps.

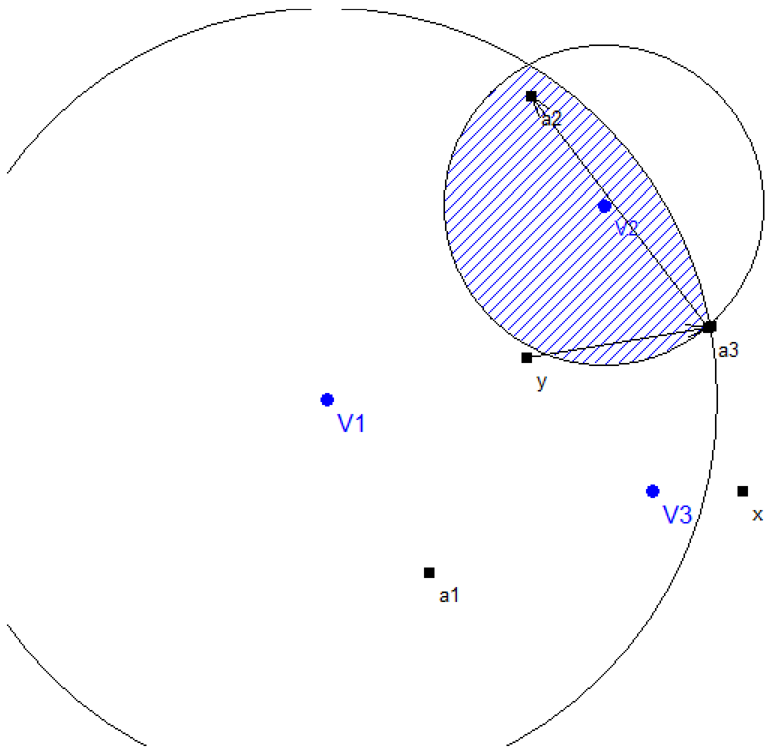
Intuition: Shifting majorities

Step	1	2	3	Coalition
a_3Py	y	a_3	a_3	V2, V3
a_2Pa_3				
a_1Pa_2				
xPa_1				



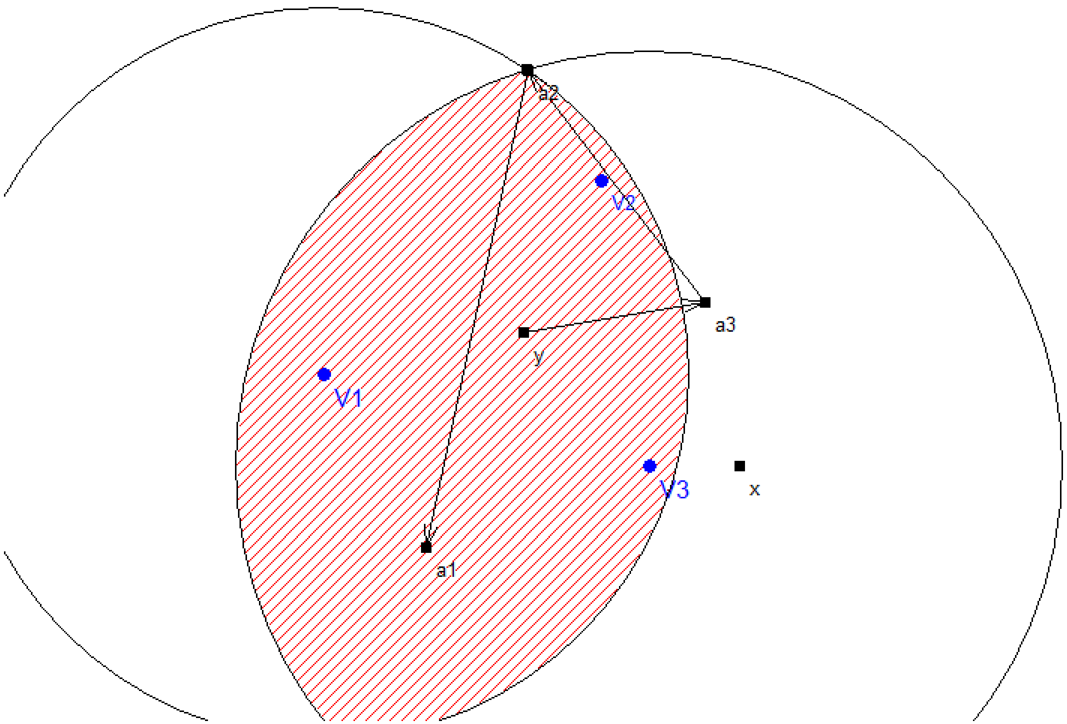
Intuition: Shifting majorities

Step	1	2	3	Coalition
a_3Py				V2, V3
a_2Pa_3	a_2	a_2	a_3	V1, V2
a_1Pa_2				
xPa_1				



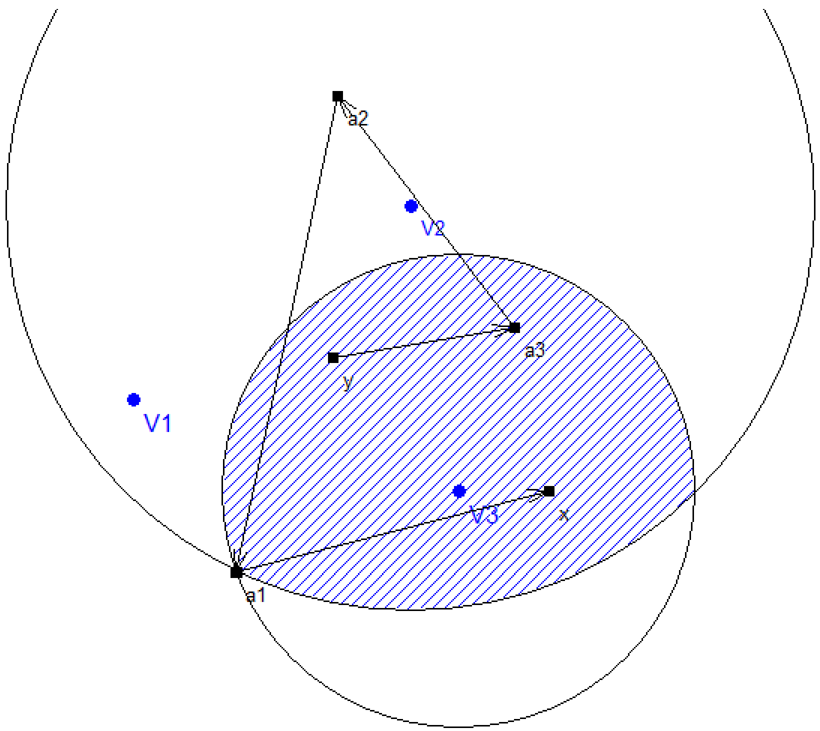
Intuition: Shifting majorities

Step	1	2	3	Coalition
a_3Py				V2, V3
a_2Pa_3				V1, V2
a_1Pa_2	a_1	a_2	a_1	V1, V3
xPa_1				



Intuition: Shifting majorities

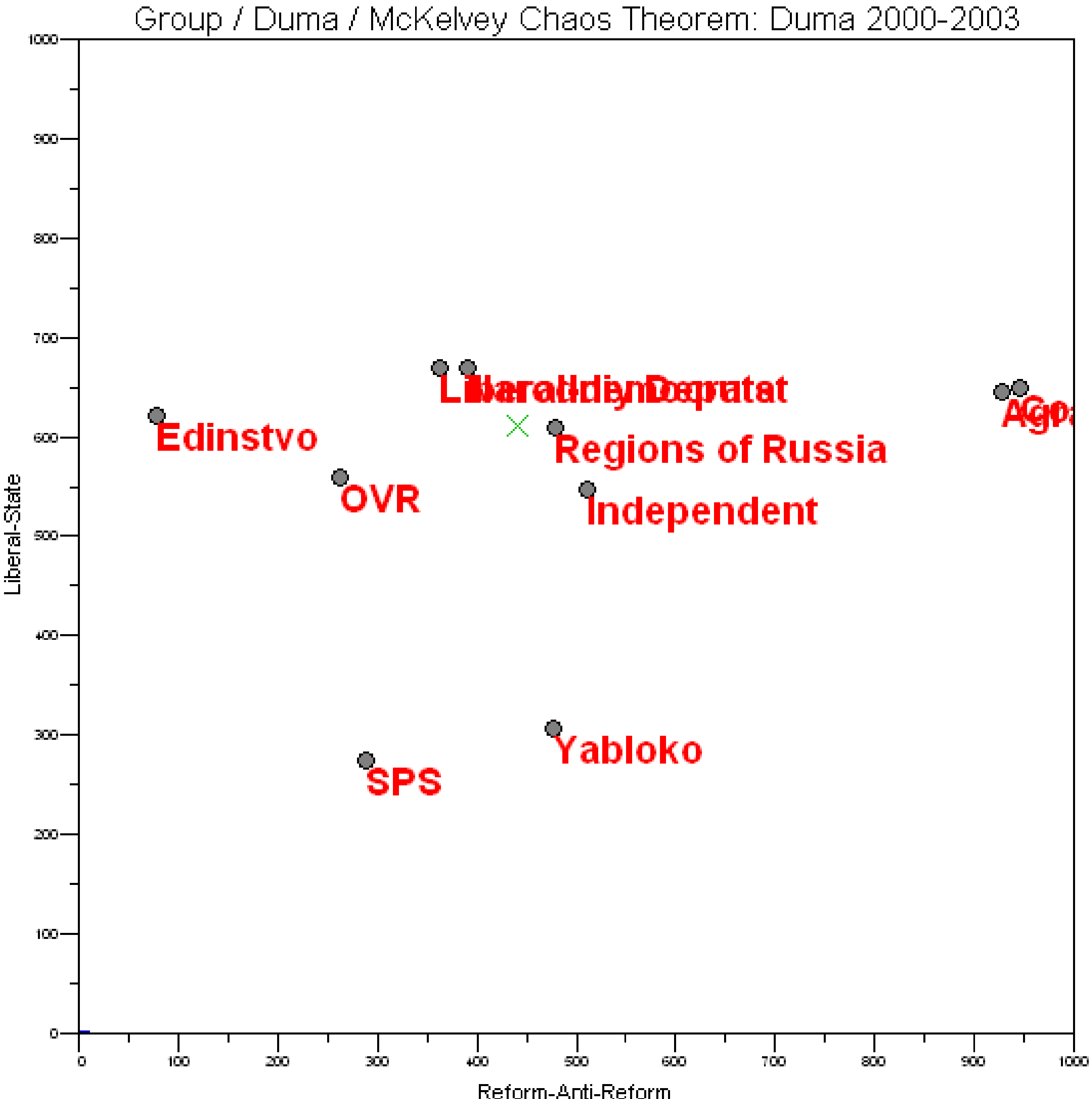
Step	1	2	3	Coalition
a_3Py				V2, V3
a_2Pa_3				V1, V2
a_1Pa_2				V1, V3
xPa_1	a_1	x	x	V2, V3



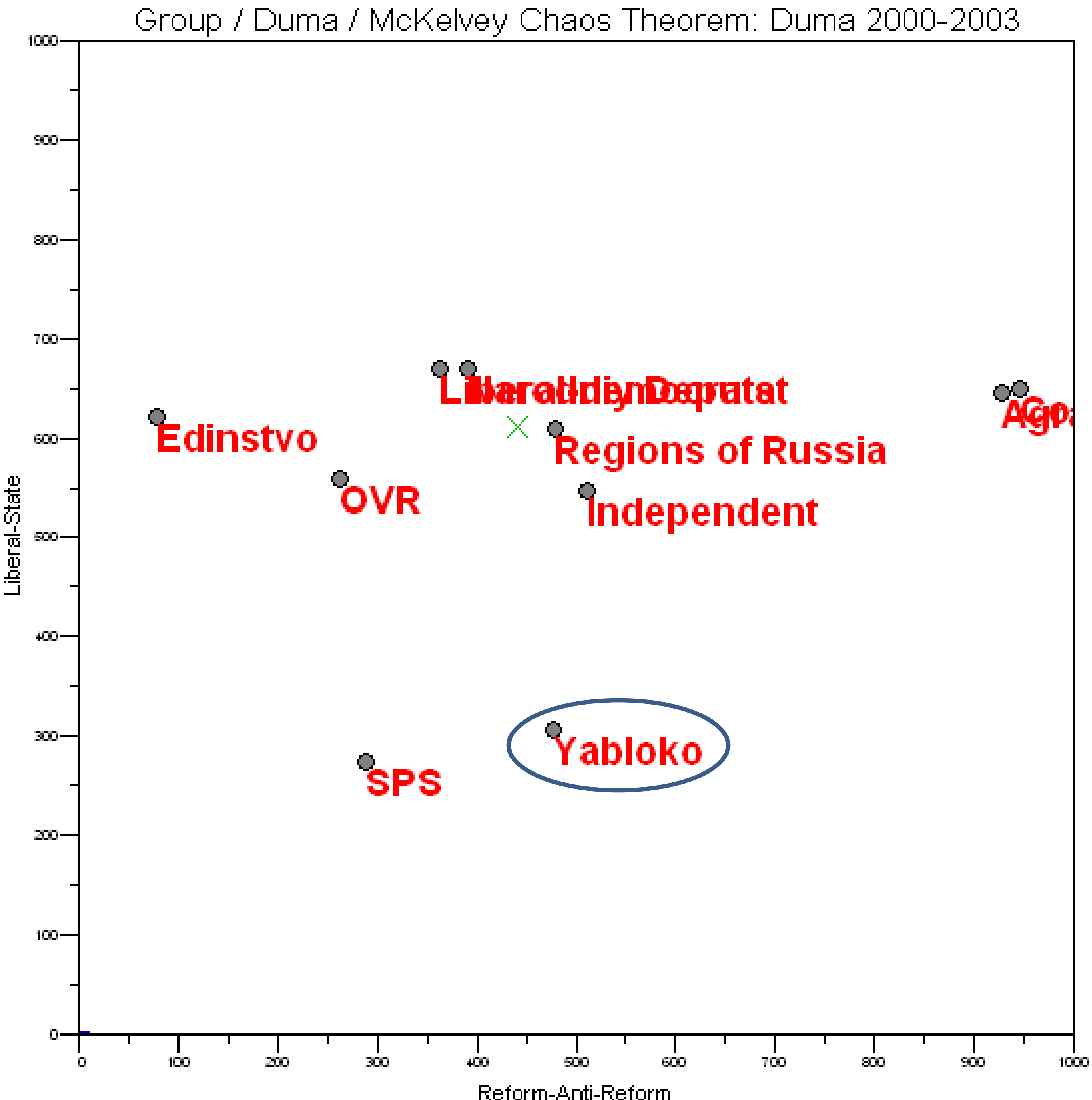
Remarks

- Even though the win set of a “centrally” located alternative is smaller than the win set of an “extreme” alternative, an agenda of pairwise majority rule votes can be constructed such that an extreme alternative emerges as the final outcome.
- In other words, with complete information an agenda setter “could” move us anywhere in the space.

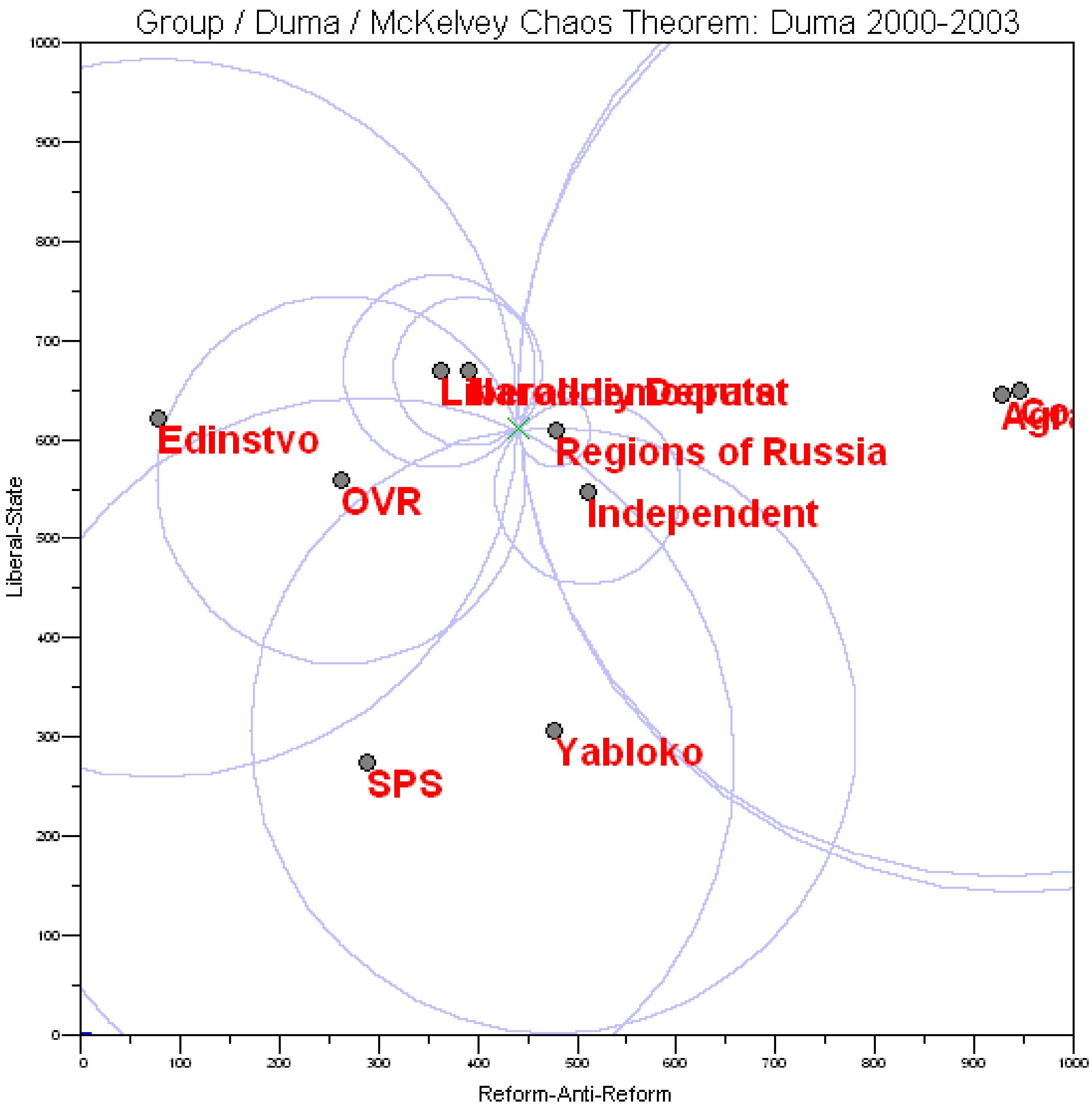
- Tips for constructing a cycle
  - Shift or alternate the majority coalition for each successive vote
  - Choose alternatives successively further away from the “middle,” thereby creating larger and larger win sets.



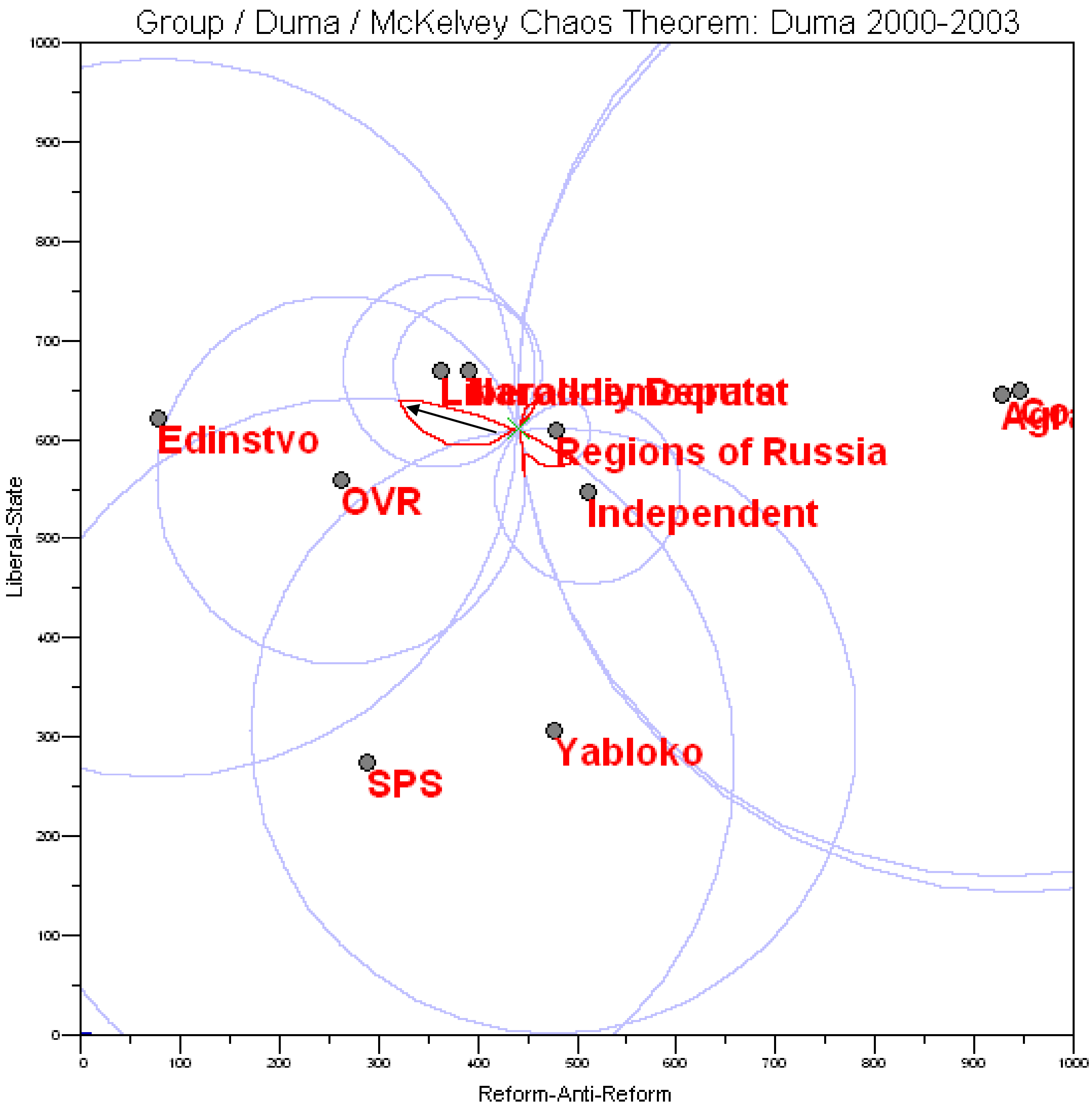
Russian Duma 2000-2003



Suppose the status quo is at x and the Yabloko gets to set the agenda.



McKelvey claimed that an agenda setter "could" move us anywhere in the space.



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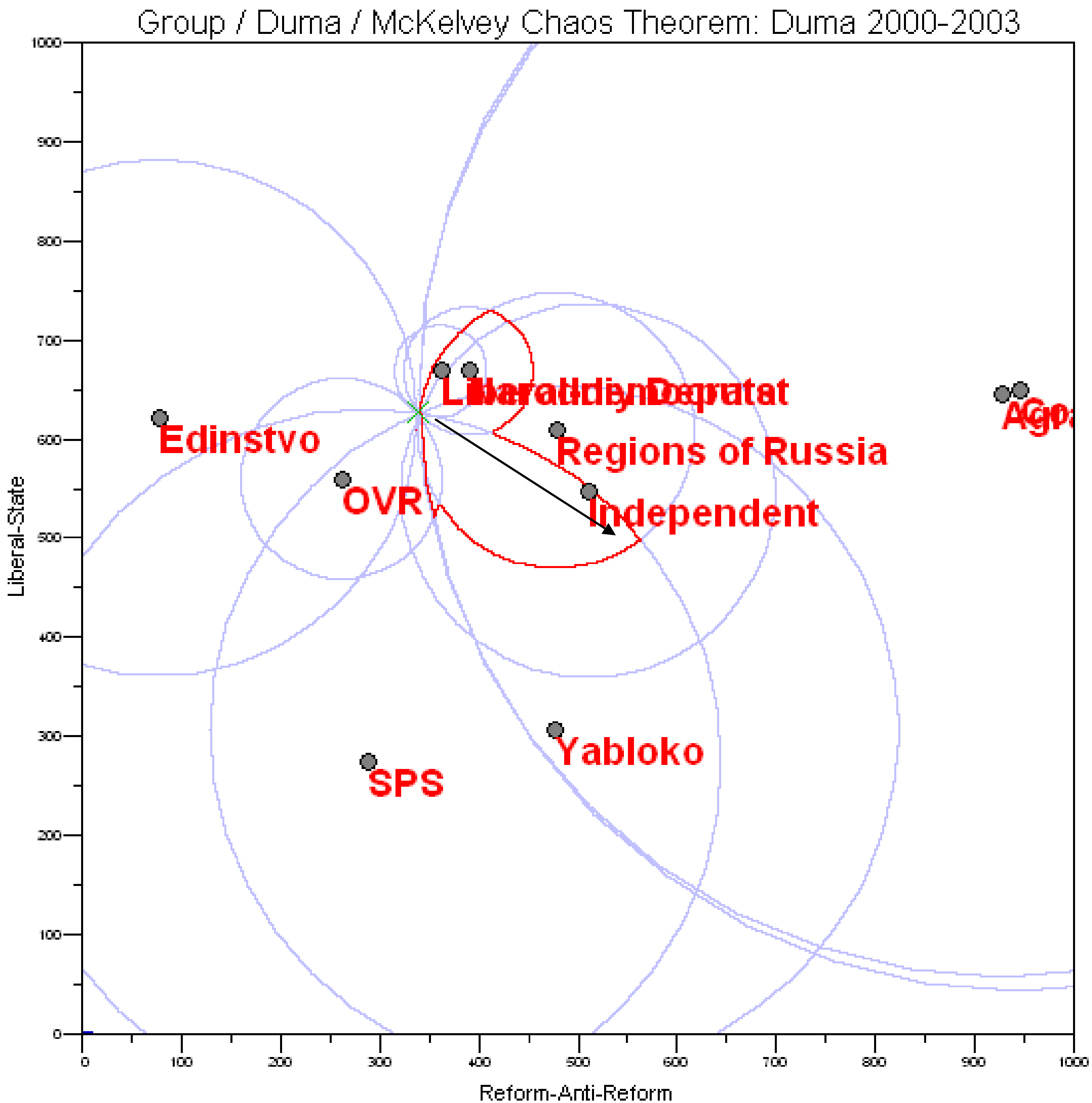

Group / Duma / McKelvey Chaos Theorem: Duma 2000-2003
Edinstvo
OVR
Regions of Russia
Independent
Yabloko
SPS
Liberal-State
Reform-Anti-Reform
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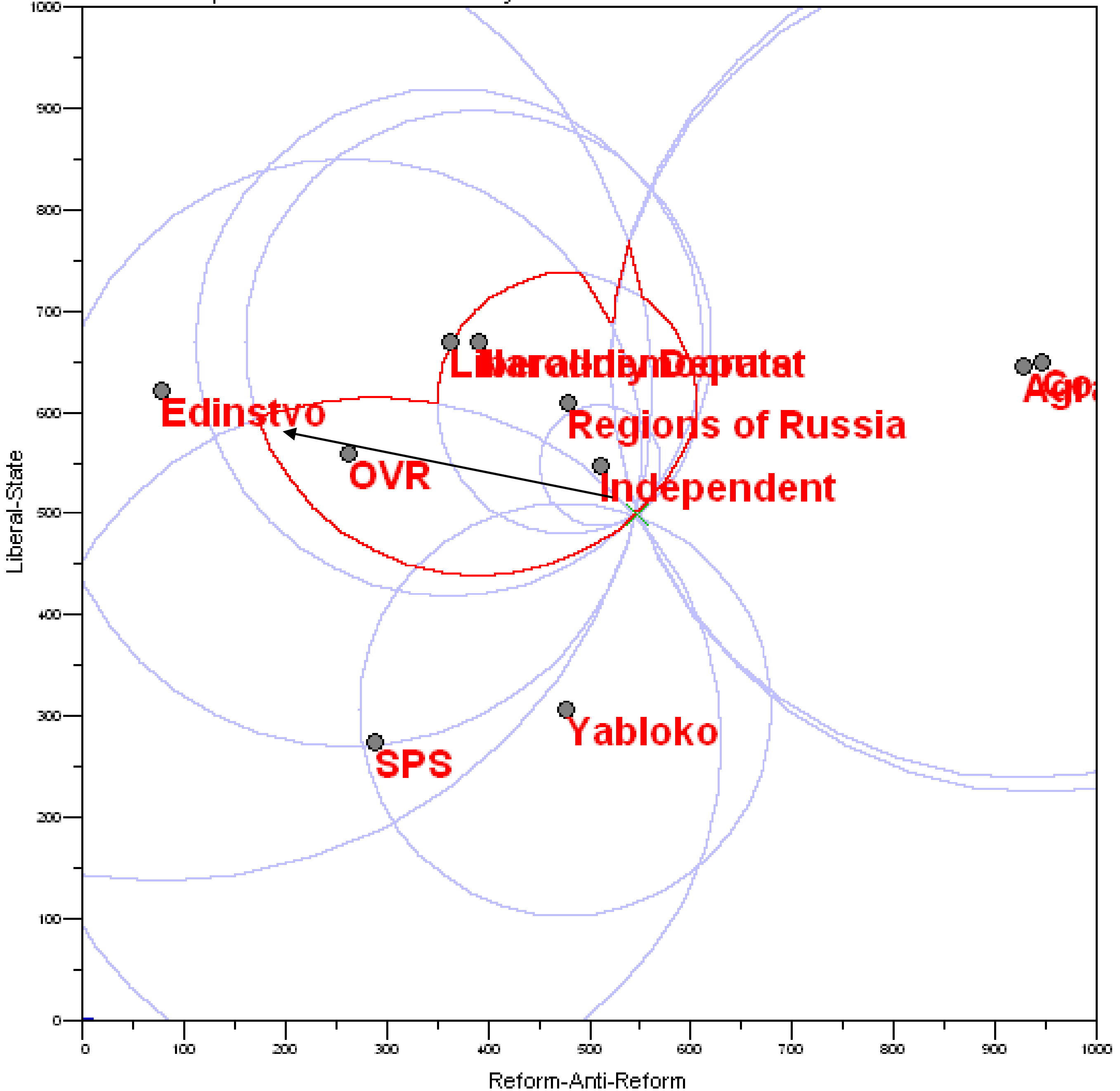
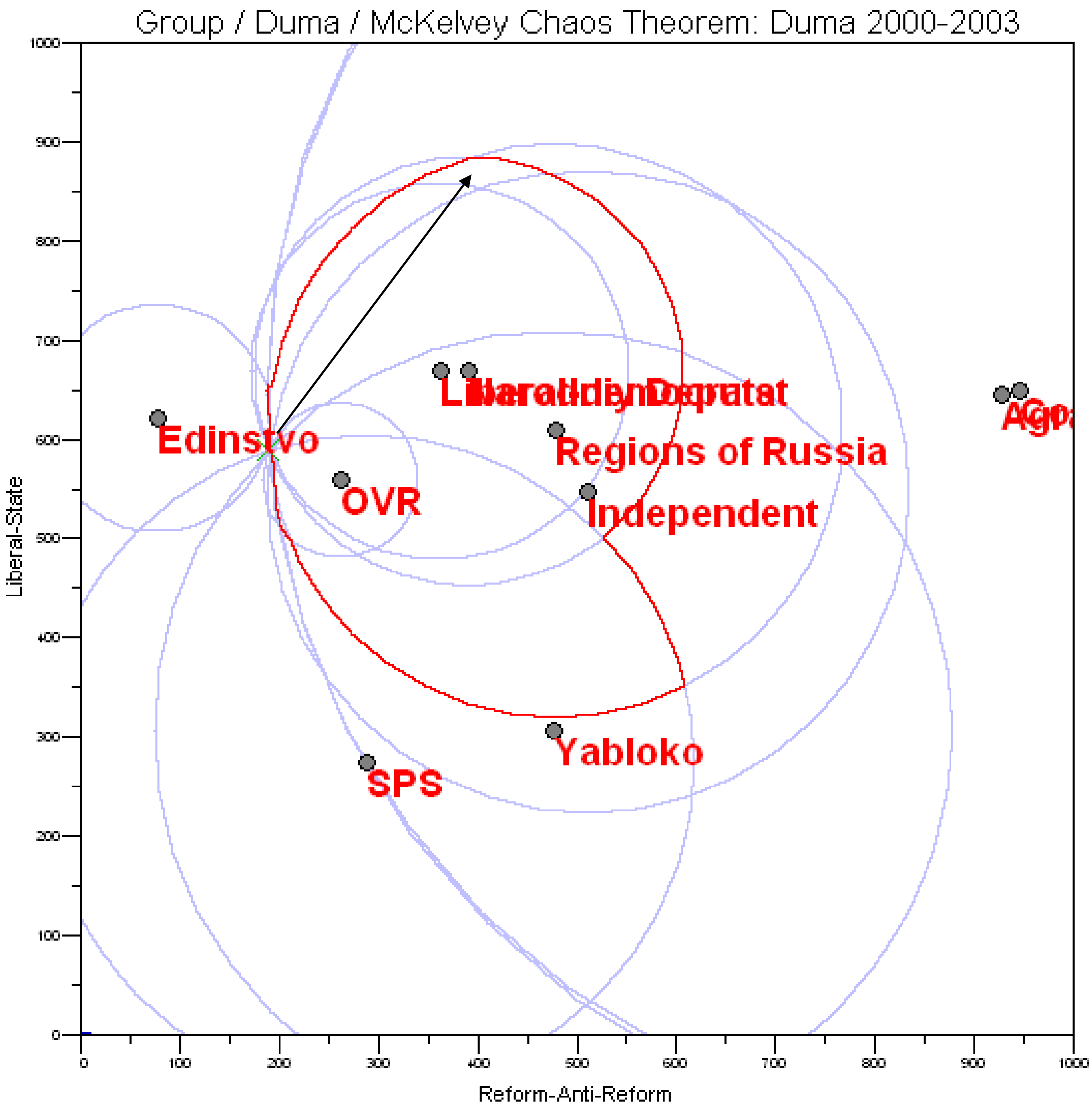
Group / Duma / McKelvey Chaos Theorem: Duma 2000-2003





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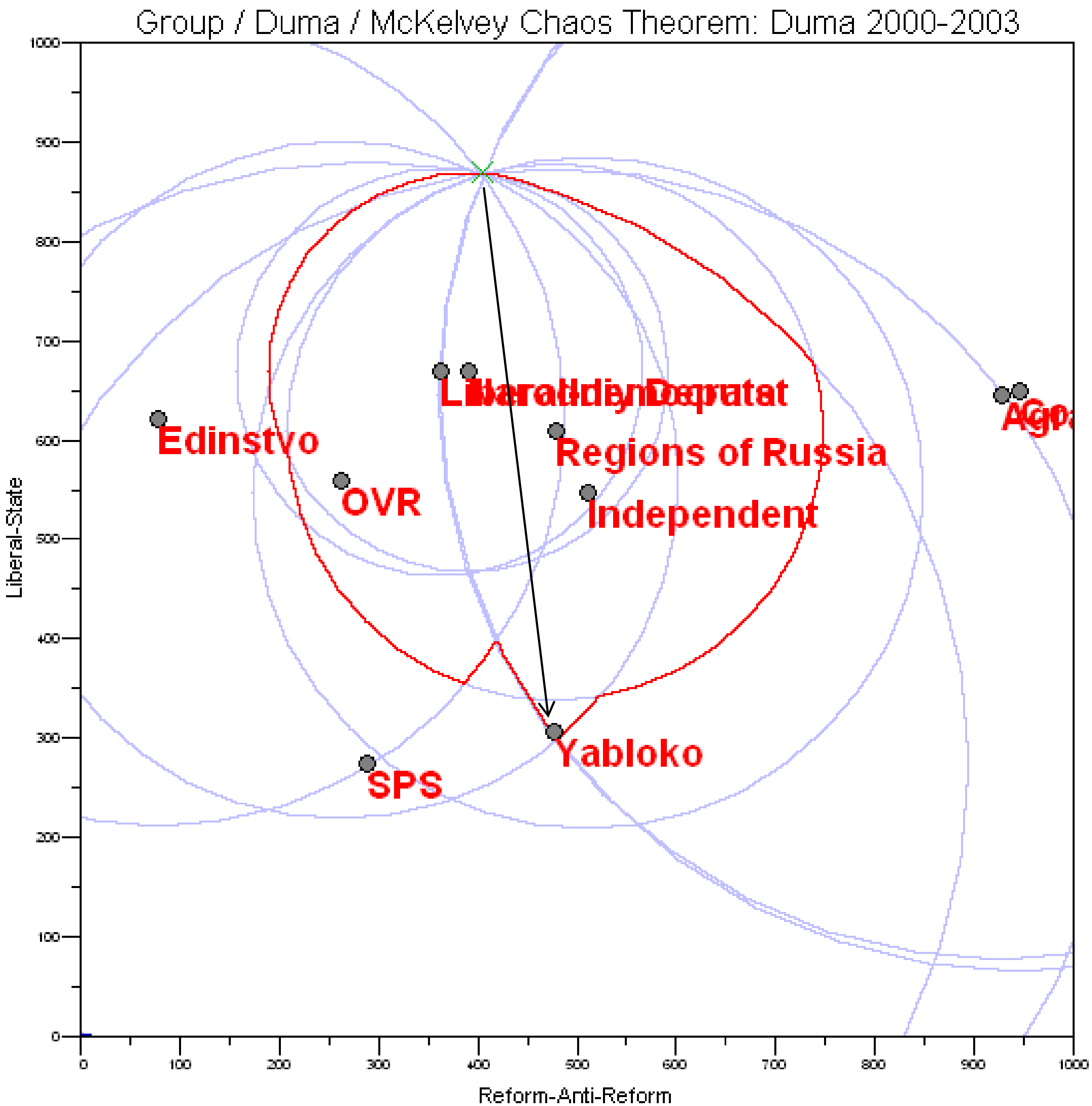

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Substantive implications

Chaos?

- Common misinterpretation (1980s “new institutionalism”)
- McKelvey’s Theorem *does not* imply “chaos.”

Agenda power?

- Suppose an individual has the power to choose the voting agenda, McKelvey’s Theorem *does imply* that the agenda setter is quite powerful.
- From any status quo y, the agenda setter can produce *any* outcome x by choosing the appropriate agenda.

Dougherty-Edward Theorem

Assume:

1. a voter is designated proposer in the last round (a variety of proposers and proposal processes can be used in earlier rounds),
2. proposals are strategic in the last round,
3. individuals vote strategically (or sincerely),
4. $R \geq 1$ rounds of voting, and
5. complete information.

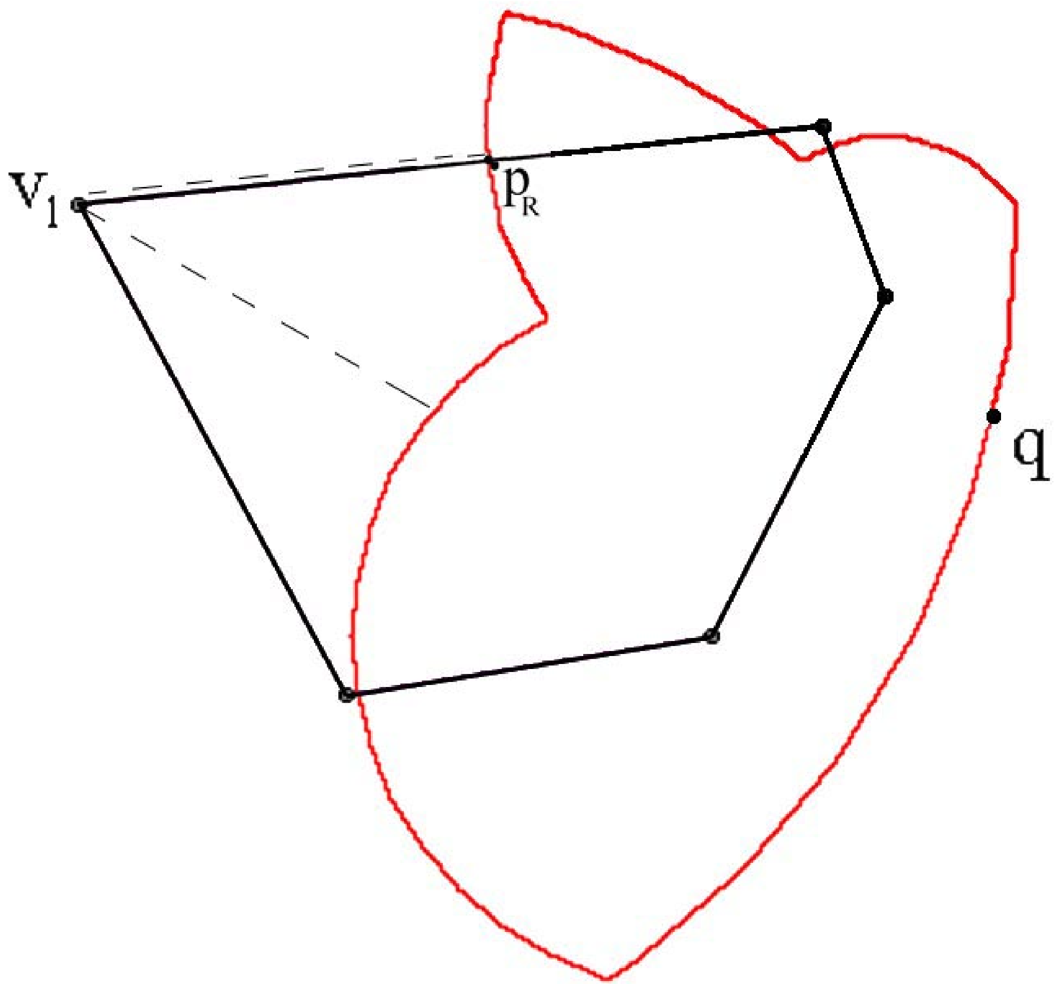
Dougherty-Edward Theorem

Theorem. Denote by q_R the status quo in the final round R. Suppose there exists a point $z \in W(q_R)$ of minimal distance to the proposer. Then given assumptions 1-5, a group using majority rule will select a Pareto optimal outcome in subgame perfect equilibrium (SPE).

Remarks

- Uses a different equilibrium concept than the core.
- The theorem shows that in finite rounds of play, a rational agenda setter will get us to an outcome in the Pareto set. He/she will not let us wonder anywhere.

Dougherty-Edward Theorem



Concluding thoughts

- Single dimension
  - The location of the median voter's ideal point is the unique element of the majority rule core.
  - Majority rule creates a transitive preference order for society that is identical to the median voter's preference order.
- Multiple dimensions. If the Plott conditions are not met,…
  - the majority rule core is empty (i.e., no equilibrium).
  - Majority rule creates an *intransitive* order for the entire set of alternatives. Hence, an agenda setter could take us anywhere.
- Dougherty-Edward (2010)
  - Even though we could go anywhere, if we model the proposal process, a rational agenda setter will bring us to an outcome in the Pareto set (i.e., equilibrium again).