Packet due Monday

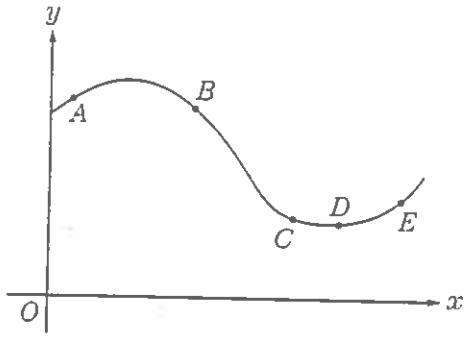
1. What is $\lim_{h \to 0} \dfrac{\cos\left(\frac{3\pi}{2} + h\right) - \cos\left(\frac{3\pi}{2}\right)}{h}$?

   (A) 1
   (B) $\frac{\sqrt{2}}{2}$
   (C) 0
   (D) -1
   (E) The limit does not exist.

$\frac{d}{dx} \cos x \Big]_{x=3\pi/2} = -\sin x \Big]_{x=\frac{3\pi}{2}} = 1$

2. At which of the five points on the graph in the figure at the right are $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ both negative?

   (A) A
   (B) B
   (C) C
   (D) D
   (E) E



3. The slope of the tangent to the curve $y^3x + y^2x^2 = 6$ at $(2, 1)$ is

   (A) $-\frac{3}{2}$
   (B) -1
   (C) $-\frac{5}{14}$
   (D) $-\frac{3}{14}$
   (E) 0

$y^3 + x \cdot 3y^2 \frac{dy}{dx} + y^2 \cdot 2x + x^2 \cdot 2y \frac{dy}{dx} = 0$

$\frac{dy}{dx}(3xy^2 + 2x^2y) = -y^3 - 2xy^2$

$\frac{dy}{dx} = \frac{-y^3 - 2xy^2}{3xy^2 + 2x^2y} \Big]_{x=2,\ y=1}$

$= \frac{-1 - 2(2)\cdot 1}{3 \cdot 2 \cdot 1 + 2 \cdot 2^2 \cdot 1} = \frac{-1-4}{6+8}$

$= -\frac{5}{14}$

4. Let S be the region enclosed by the graphs of $y = 2x$ and $y = 2x^2$ for $0 \le x \le 1$. What is the volume of the solid generated when S is revolved about the line $y = 3$?

(A) $\pi \int_0^1 \left((3 - 2x^2)^2 - (3 - 2x)^2 \right) dx$

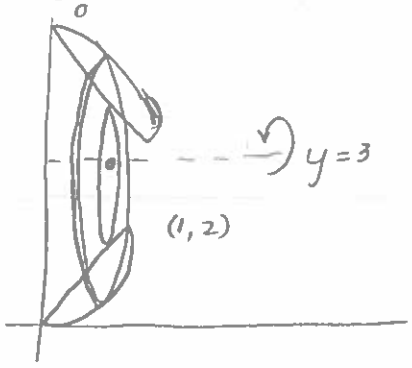
(B) $\pi \int_0^1 \left((3 - 2x)^2 - (3 - 2x^2)^2 \right) dx$

(C) $\pi \int_0^1 \left(4x^2 - 4x^4 \right) dx$

(D) $\pi \int_0^2 \left(\left(3 - \frac{y}{2}\right)^2 - \left(3 - \sqrt{\frac{y}{2}}\right)^2 \right) dy$

(E) $\pi \int_0^2 \left(\left(3 - \sqrt{\frac{y}{2}}\right)^2 - \left(3 - \frac{y}{2}\right)^2 \right) dy$

Vol = $\pi \int_0^1 \left[(3 - 2x^2)^2 - (3 - 2x)^2 \right] dx$



5. Which of the following statements about the function given by $f(x) = x^4 - 2x^3$ is true?

$f'(x) = 4x^3 - 6x^2 = 2x^2(2x - 3)$ CP $x = 0, y = 3/2$

$= x^3(x - 2)$

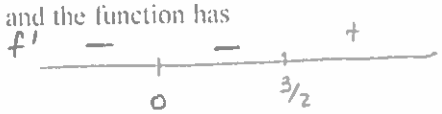
(A) The function has no relative extremum.

(B) The graph of the function has one point of inflection and the function has two relative extrema.

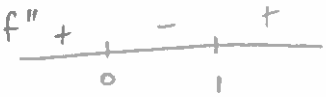
(C) The graph of the function has two points of inflection and the function has one relative extremum.

(D) The graph of the function has two points of inflection and the function has two relative extrema.

(E) The graph of the function has two points of inflection and the function has three relative extrema.



$f'' = 12x^2 - 12x = 12x(x - 1)$



6. If $f(x) = \sin^2(3 - x)$, then $f'(0) =$

(A) $-2 \cos 3$

(B) $-2 \sin 3 \cos 3$

(C) $6 \cos 3$

(D) $2 \sin 3 \cos 3$

(E) $6 \sin 3 \cos 3$

$f'(x) = 2 \sin(3 - x) \cdot \cos(3 - x)(-1)$

$f'(0) = -2 \sin 3 \cos 3$

7. Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{4x}{y}$, where $y(2) = -2$?

(A) $y = 2x$ for $x > 0$

(B) $y = 2x - 6$ for $x \ne 3$

(C) $y = -\sqrt{4x^2 - 12}$ for $x > \sqrt{3}$

(D) $y = \sqrt{4x^2 - 12}$ for $x > \sqrt{3}$

(E) $y = -\sqrt{4x^2 - 6}$ for $x > \sqrt{1.5}$

$\int_{-2}^{y} y \, dy = \int_{2}^{x} 4x \, dx$

$\frac{y^2}{2} \Big]_{-2}^{y} = 2x^2 \Big]_{2}^{x}$

$\frac{y^2}{2} - 2 = 2x^2 - 8$

$y^2 - 4 = 4x^2 - 16$

$y^2 = 4x^2 - 12$

$y = -\sqrt{4x^2 - 12}$

$4x^2 - 12 \ge 0$

$4x^2 \ge 12$

$x^2 \ge 3 \quad |x| \ge \sqrt{3}$

8. What is the average rate of change of the function f given by $f(x) = x^4 - 5x$ on the closed interval $[0, 3]$?

(A) 8.5
(B) 8.7
(C) 22
(D) 33
(E) 66

$f(3) = 3^4 - 5(3) = 81 - 15 = 66$

$f(0) = 0$

avg rate of change $= \frac{f(3) - f(0)}{3 - 0} = \frac{66}{3} = 22$

9. The position of a particle moving along a line is given by $s(t) = 2t^3 - 24t^2 + 90t + 7$ for $t \geq 0$. For what values of t is the speed of the particle increasing?

(A) $3 < t < 4$ only
(B) $t > 4$ only
(C) $t > 5$ only
(D) $0 < t < 3$ and $t > 5$
(E) $3 < t < 4$ and $t > 5$

speed increases when velocity and accel have the same sign

$v(t) = 6t^2 - 48t + 90$

$6(t^2 - 8t + 15) = 0$

$(t - 5)(t - 3) = 0$

$t = 5 \quad t = 3$

✓ + | − | + (3, 5)

$a(t) = 12t - 48 = 0$

$t = 4$

a: − | + (4)

10. $\int (x - 1)\sqrt{x}\, dx =$

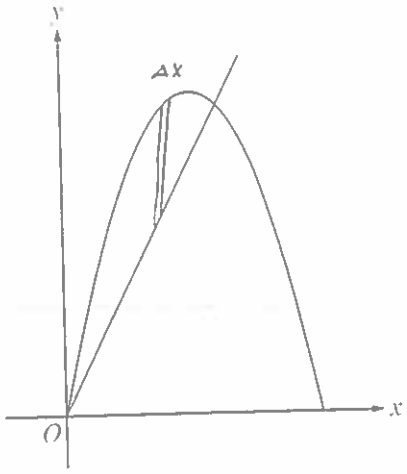
(A) $\frac{3}{2}\sqrt{x} - \frac{1}{\sqrt{x}} + C$
(B) $\frac{2}{3}x^{3/2} + \frac{1}{2}x^{1/2} + C$
(C) $\frac{1}{2}x^2 - x + C$
(D) $\frac{2}{5}x^{5/2} - \frac{2}{3}x^{3/2} + C$
(E) $\frac{1}{2}x^2 + 2x^{3/2} - x + C$

$\int (x-1)\, x^{1/2}\, dx = \int (x^{3/2} - x^{1/2})\, dx$

$= \frac{2}{5}x^{5/2} - \frac{2}{3}x^{3/2} + C$

11. What is $\lim_{x \to \infty} \frac{x^2 - 4}{2 + x - 4x^2}$?

(A) -2
(B) $-\frac{1}{4}$
(C) $\frac{1}{2}$
(D) 1
(E) The limit does not exist.



$5x - x^2 = 2x$

$x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0 \quad x = 3$

12. The figure above shows the graph of $y = 5x - x^2$ and the graph of the line $y = 2x$. What is the area of the shaded region?

(A) $\frac{25}{6}$

(B) $\frac{9}{2}$

(C) 9

(D) $\frac{27}{2}$

(E) $\frac{45}{2}$

$$\int_0^3 (5x - x^2 - 2x)\,dx$$

$$= \int_0^3 (3x - x^2)\,dx = \frac{3x^2}{2} - \frac{x^3}{3}\Big]_0^3$$

$$= \frac{27}{2} - 9 = \frac{9}{2}$$

13. If $y = 5 + \int_2^{2x} e^{-t^2}\,dt$, which of the following is true?

(A) $\frac{dy}{dx} = e^{-x^2}$ and $y(0) = 5$

(B) $\frac{dy}{dx} = e^{-x^2}$ and $y(1) = 5$

(C) $\frac{dy}{dx} = e^{-4x^2}$ and $y(1) = 5$

(D) $\frac{dy}{dx} = 2e^{-4x^2}$ and $y(0) = 5$

(E) $\frac{dy}{dx} = 2e^{-4x^2}$ and $y(1) = 5$

$$\frac{dy}{dx} = \frac{d}{dx}\int_2^{2x} e^{-t^2}\,dt$$

$$= e^{-(2x)^2} \cdot 2$$

$$= 2e^{-4x^2}$$

$$y(0) = 5 + \int_2^0 ?$$

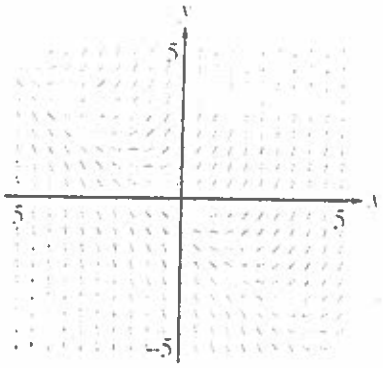
$$y(1) = 5 + \int_2^2 \sim$$

$$\therefore y = 0$$

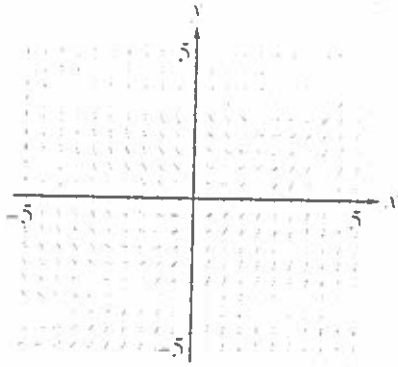
14. Which of the following is a slope field for the differential equation $\frac{dy}{dx} = \frac{x}{y}$?

Quad I +
II −
III +
IV −

(A)



(B)



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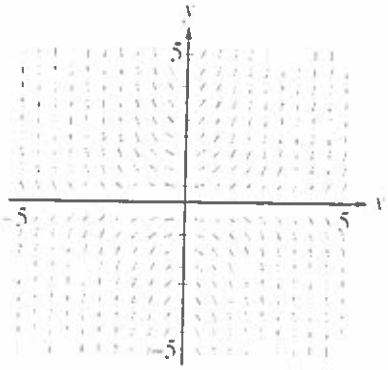

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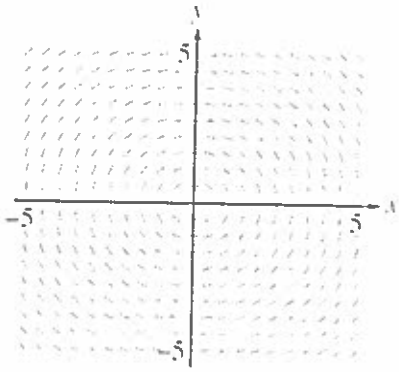

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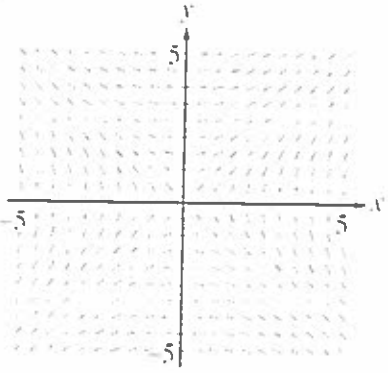


$$\frac{dy}{dx} = \frac{x}{y}$$

$$\int y\,dy = \int x\,dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + C$$

$$y^2 = x^2 + C$$

$$y = \pm\sqrt{x^2 + C}$$

Part B Sample Multiple-Choice Questions

A graphing calculator is required for some questions on this part of the exam.

Part B consists of 17 questions. In this section of the exam, as a correction for guessing, one-fourth of the number of questions answered incorrectly will be subtracted from the number of questions answered correctly. Following are the directions for Section I, Part B, and a representative set of 10 questions.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

15. A particle travels along a straight line with a velocity of $v(t) = 3e^{(-t/2)}\sin(2t)$ meters per second. What is the total distance, in meters, traveled by the particle during the time interval $0 \le t \le 2$ seconds?

(A) 0.835
(B) 1.850
(C) 2.055
(D) 2.261
(E) 7.025

$dist = \int |v(t)|\,dt = \int_0^2 |3e^{-t/2}\sin(2t)|\,dt$

16. A city is built around a circular lake that has a radius of 1 mile. The population density of the city is $f(r)$ people per square mile, where r is the distance from the center of the lake, in miles. Which of the following expressions gives the number of people who live within 1 mile of the lake?

(A) $2\pi\int_0^1 r f(r)\,dr$

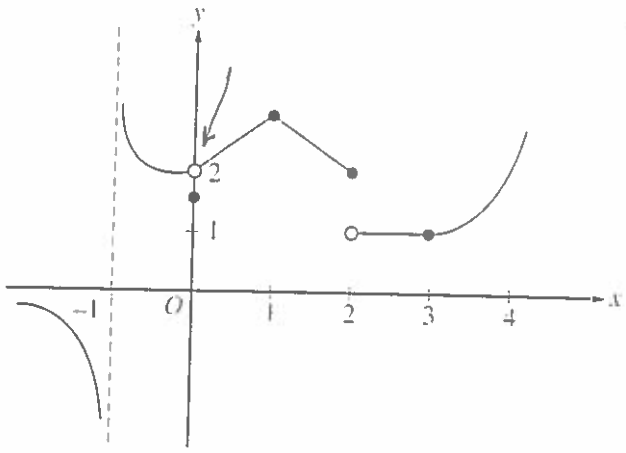
(B) $2\pi\int_0^1 r(1 + f(r))\,dr$

D

(C) $2\pi\int_0^2 r(1 + f(r))\,dr$

(D) $2\pi\int_1^2 r f(r)\,dr$

(E) $2\pi\int_1^2 r(1 + f(r))\,dr$



17. The graph of a function f is shown above. If $\lim_{x \to b} f(x)$ exists and f is not continuous at b, then $b =$

(A) -1
(B) 0
(C) 1
(D) 2
(E) 3

$-.02$ $-.01$ $\div .1$
$.20$ $.18$ $.17$

x	1.1	1.2	1.3	1.4
$f(x)$	4.18	4.38	4.56	4.73

18. Let f be a function such that $f''(x) < 0$ for all x in the closed interval $[1, 2]$. Selected values of f are shown in the table above. Which of the following must be true about $f'(1.2)$?

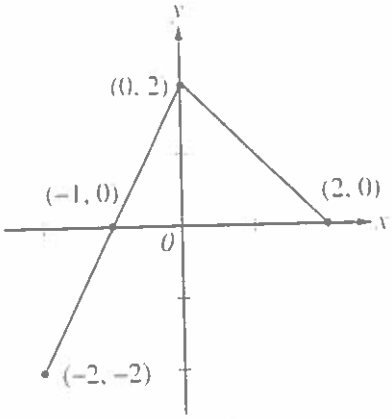
(A) $f'(1.2) < 0$
(B) $0 < f'(1.2) < 1.6$
(C) $1.6 < f'(1.2) < 1.8$
(D) $1.8 < f'(1.2) < 2.0$
(E) $f'(1.2) > 2.0$

19. Two particles start at the origin and move along the x-axis. For $0 \le t \le 10$, their respective position functions are given by $x_1 = \sin t$ and $x_2 = e^{-2t} - 1$. For how many values of t do the particles have the same velocity?

(A) None
(B) One
(C) Two
(D) Three
(E) Four

graph
$v_1 = \cos t$
$v_2 = -2e^{-2t}$
how many times from $0 \le t \le 10$ do graphs intersect?

graph + trace bottom curve



Graph of f

20. The graph of the function f shown above consists of two line segments. If g is the function defined by $g(x) = \int_0^x f(t)\,dt$, then $g(-1) =$

(A) -2
(B) -1
(C) 0
(D) 1
(E) 2

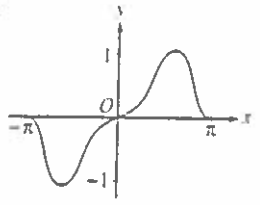
$$g(-1) = \int_0^{-1} f(t)\,dt$$

$$= -\int_{-1}^{0} f(t)\,dt$$

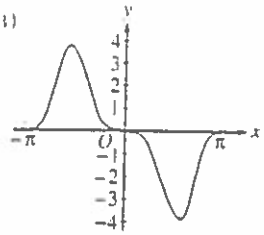
$$= -\frac{1}{2}(1)(2) = -1$$

21. The graphs of five functions are shown below. Which function has a nonzero average value over the closed interval $[-\pi, \pi]$?

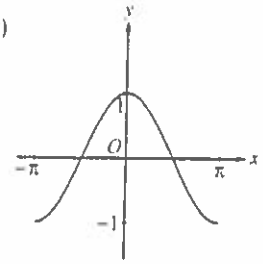
(A)



(B)



(C)



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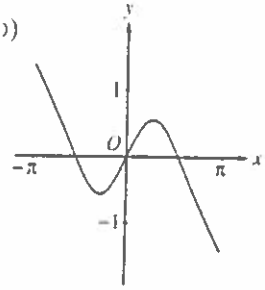

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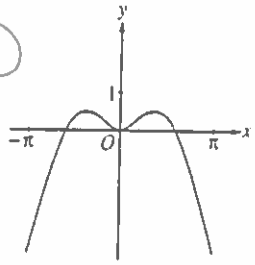

22. A differentiable function f has the property that $f(5) = 3$ and $f'(5) = 4$. What is the estimate for $f(4.8)$ using the local linear approximation for f at $x = 5$?

(A) 2.2
(B) 2.8
(C) 3.4
(D) 3.8
(E) 4.6

$y - 3 = 4(x-5)$

$y = 4(x-5) + 3$

$f(4.8) \approx 4(4.8-5) + 3$

$= 4(-.2) + 3$

$= -.8 + 3 = 2.2$

23. Oil is leaking from a tanker at the rate of $R(t) = 2{,}000e^{-0.2t}$ gallons per hour, where t is measured in hours. How much oil leaks out of the tanker from time $t = 0$ to $t = 10$?

$$\int_0^{10} 2000\, e^{-.2t}\, dt =$$

(A) 54 gallons
(B) 271 gallons
(C) 865 gallons
(D) 8,647 gallons
(E) 14,778 gallons

24. If $f'(x) = \sin\left(\frac{\pi e^x}{2}\right)$ and $f(0) = 1$, then $f(2) =$

$$f(2) = f(0) + \int_0^2 f'(x)\, dx$$

$$= 1 + .157$$

(A) -1.819
(B) -0.843
(C) -0.819
(D) 0.157
(F) 1.157